European Water 57: 505-512, 2017.

Groundwater flow simulation in confined aquifer by meshless element free Galerkin method

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Abstract: In groundwater management problems, the flow behaviour of the aquifer system is analysed by solving the governing equations analytically or using numerical techniques. Numerical techniques such as finite difference method (FDM) and finite element method (FEM) are in use for many decades for solving complex flow problems as these cannot be addressed by analytical methods. In the recent years, meshless methods have been effectively used in solving the groundwater problems. These methods do not require mesh formation resulting in lesser computational cost and time as compared to mesh based methods. The element free Galerkin (EFG) method is one of the efficient and accurate meshless method for the complex flow problems. This method uses moving least square (MLS) approximation for obtaining the shape functions which lack the Kronecker delta function property. Therefore, presently the Dirichlet boundary conditions are imposed using the Lagrange multiplier method. The present study simulates the groundwater flow in a hypothetical heterogeneous and anisotropic confined aquifer using EFG method. The aquifer system is imposed with both Dirichlet and Neumann boundary conditions. The numerical outputs of the EFG model in MATLAB are verified with the groundwater head values obtained from the FEM simulation. The satisfactory results of the EFG model shows the efficacy of the present method.

Key words: Element free Galerkin method, groundwater flow simulation, meshless method, moving least square approximation

1. INTRODUCTION

Groundwater is relatively pure and a convenient source of drinking water in many parts of the world. The increasing population and rapid growth of industrial sectors worldwide have raised the major problems of depletion of the groundwater table and contamination of the groundwater resources. The solution to these problems demands for proper management of the groundwater resources. Groundwater modeling plays a crucial role in predicting the spatial and temporal distribution of the groundwater in the aquifer system with the help of available analytical and numerical techniques. The analytical solutions are based on fundamental laws of nature and thus have been used only for obtaining the solution of simplified groundwater flow problems. The mesh based numerical techniques of FDM and FEM have been extensively used in solving the complex groundwater problems. These methods initially divides the whole problem domain into meshes/grids or elements connected together by field nodes in a well-defined manner (Liu and Gu 2005). This mesh formation procedure leads to higher computational cost and time in simulating the groundwater flow in aquifer systems. In last few decades, new methods which do not require meshing have been developed and are known as meshless methods. In these methods, the field nodes are scattered on the problem domain and its boundaries to represent the whole problem. Also, there is no connectivity among the field nodes as prerequisite in meshbased methods.

The application of meshless methods in groundwater problems have been used in very few studies. Park and Leap (2000) presented the solution procedure of the meshless EFG method for modeling the groundwater flow in confined aquifer domains. In their study the Dirichlet boundary conditions has been imposed using the coupled EFG-FEM technique as MLS approximation in EFG model do not satisfy Kronecker delta property. Praveen Kumar and Dodagoudar (2009) used the EFG method for estimating the contaminant distribution in the unsaturated zones of porous media. The EFG outputs were verified with the published experimental results and FEM solutions. Meenal and Eldho (2011) used the meshless Point Collocation method (PCM) with multiquadric radial basis function for simulating the groundwater flow in unconfined aquifers. The efficacy of the developed model was ensured by verifying its outputs with analytical and FEM solutions. Swathi and Eldho (2014) used the Galerkin equivalent of Meshless Local Petrov-Galerkin (MLPG) method with exponential radial basis function for solving the unconfined aquifer problems. The results of this method on comparison with available analytical and numerical solutions were found to be satisfactory.

The EFG model developed by Belytschko et al. (1994) employs the use of MLS approximation for approximating the unknown function and thus, the Dirichlet boundary conditions requires special treatment. In the available literature, the existing techniques for implementing the Dirichlet boundary conditions in EFG method includes Lagrange multiplier method (Belytschko et al. 1994), modified variational principles (Lu et al. 1994) and coupled EFG-FEM technique (Belytschko et al. 1995). The results obtained by using Lagrange multipliers are very accurate (Park and Leap 2000). Therefore in the present study the Lagrange multiplier method is used to implement the Dirichlet boundary conditions. The main objective of this study is to test the efficacy of the EFG method in simulating the groundwater flow in a confined aquifer system by comparing its outputs with FEM solutions.

2. ELEMENT FREE GALERKIN METHOD

The EFG method is a meshless method based on the Galerkin weak form defined over the global problem domain (Liu and Gu 2005). This technique uses MLS approximation in the Galerkin weak form to develop a set of linear algebraic equations. It also requires a set of background cells for performing the numerical integration of integral form obtained from the Galerkin method. It is performed using the Gauss quadrature scheme over these cells. The present study uses the cubic spline weight function for evaluating the EFG shape functions from MLS approximation of unknown head. The EFG shape functions and its derivative are computed using the procedure given in Praveen Kumar and Dodagoudar (2009).

2.1 EFG discretization of governing equation

The governing equation for the two dimensional groundwater flow in a heterogeneous and anisotropic confined aquifer is given as (Bear 1979)

$$\frac{\partial}{\partial x}\left[T_x \frac{\partial h}{\partial x}\right] + \frac{\partial}{\partial y}\left[T_y \frac{\partial h}{\partial y}\right] = S\frac{\partial h}{\partial t} + Q_w\,\delta(x - x_i)(y - y_j) - q \qquad (1)$$

The initial conditions used are written as

$$h(x, y, 0) = h_0\,(x, y) \quad x, y \in \Omega \qquad (2)$$

The boundary conditions used for the groundwater flow problems are represented as

$$h(x, y, t) = h_D\,(x, y, t) \qquad x, y \in \Gamma_S \qquad (3)$$

$$T\frac{\partial h}{\partial n} = q_B\,(x, y, t) \qquad x, y \in \Gamma_E \qquad (4)$$

where $h(x, y, t)$ is the piezometric head (m), T_x and T_y are the transmissivities in x and y directions respectively (m^2/d), S is the storage coefficient, Q_w is a source or sink function ($-Q_w$= source, + Q_w= sink) (m^3/d/m^2), δ is the Dirac delta function with the property that when $x = x_i$ and $y = y_j$

then $\delta = 1$ otherwise 0, Ω is the flow region, Γ is the boundary region ($\Gamma_S \cup T_E = \Gamma$), $\partial/\partial n$ is the normal derivative, q is the known recharge rate (m^3/d/m^2), $h_0(x,y)$ is the initial head in flow region (m), $h_D\ (x,y,t)$ is the known head value at the Dirichlet boundary (m), $q_B(x,y,t)$ is the known inflow rate at the Neumann boundary (m^3/d/m^2).

In EFG method initially the trial solutions $\hat{h}(x,y,t)$ is defined as $\hat{h}(x,y,t) = \sum_{I=1}^{n} h_I(t)\,\phi_I(x,y)$, where h_I and ϕ_I are the unknown head and EFG shape function at node I respectively, and n is the number of nodes in the domain of influence of point of interest.

The weak integral form of the governing groundwater flow equation [Eq. (1)] is represented as (Praveen Kumar and Dodagoudar 2009)

$$\int_{\Omega} \left(\frac{\partial}{\partial x} \left[T_x \frac{\partial h}{\partial x} \right] + \frac{\partial}{\partial y} \left[T_y \frac{\partial h}{\partial y} \right] - S \frac{\partial h}{\partial t} - Q_w + q \right) \delta h^T \; d\Omega = 0 \tag{5}$$

Using Green's theorem, Eq. (5) can be simplified to following expression

$$\int_{\Gamma_E} \left(T_x \frac{\partial h}{\partial x} n_x \right) \delta h^T \, d\Omega - \int_{\Omega} \left(T_x \frac{\partial h}{\partial x} \delta \left(\frac{\partial h^T}{\partial x} \right) \right) d\Omega + \int_{\Gamma_E} \left(T_y \frac{\partial h}{\partial y} n_y \right) \delta h^T d\Omega - \int_{\Omega} \left(T_y \frac{\partial h}{\partial y} \delta \left(\frac{\partial h^T}{\partial x} \right) \right) d\Omega$$
$$- \int_{\Omega} S \frac{\partial h}{\partial t} \delta h^T \, d\Omega - \int_{\Omega} Q_w \delta h^T \, d\Omega + \int_{\Omega} q \delta h^T \, d\Omega = 0 \tag{6}$$

On rearranging the terms in Eq. (6)

$$\int_{\Omega} \left(T_x \frac{\partial h}{\partial x} \delta \left(\frac{\partial h^T}{\partial x} \right) \right) d\Omega + \int_{\Omega} \left(T_y \frac{\partial h}{\partial y} \delta \left(\frac{\partial h^T}{\partial x} \right) \right) d\Omega + \int_{\Omega} S \frac{\partial h}{\partial t} \delta h^T \, d\Omega$$
$$= \int_{\Gamma_E} \left(T_x \frac{\partial h}{\partial x} n_x + T_y \frac{\partial h}{\partial y} n_y \right) \delta h^T \, d\Omega \; - \int_{\Omega} Q_w \delta h^T \, d\Omega + \int_{\Omega} q \delta h^T \, d\Omega \tag{7}$$

On using the Lagrange multiplier method for implementing the Dirichlet boundary conditions and Eq. (4) in Eq. (7), it can be written as

$$\int_{\Omega} \left(T_x \frac{\partial h}{\partial x} \delta \left(\frac{\partial h^{\mathrm{T}}}{\partial x} \right) \right) d\Omega + \int_{\Omega} \left(T_y \frac{\partial h}{\partial y} \delta \left(\frac{\partial h^{\mathrm{T}}}{\partial x} \right) \right) d\Omega + \int_{\Omega} S \frac{\partial h}{\partial t} \delta h^{\mathrm{T}} \, d\Omega$$
$$= q_B \int_{\Gamma_E} \delta h^{\mathrm{T}} \, d\Omega - \int_{\Omega} Q_w \delta h^{\mathrm{T}} \, d\Omega + \int_{\Omega} q \delta h^{\mathrm{T}} \, d\Omega \; - \int_{\Gamma_S} \delta \lambda^{\mathrm{T}} \, (h - h_D) d\Gamma - \int_{\Gamma_S} \lambda \, \delta h^{\mathrm{T}} \, d\Gamma \tag{8}$$

where λ is a Lagrange multiplier for incorporating the Dirichlet boundary conditions and it is defined as (Liu and Gu 2005)

$$\lambda = \sum_{K=1}^{n_\lambda} N_K(s) \lambda_K \qquad x, y \; \in \Gamma_S \tag{9}$$

where n_λ is the number of nodes used for this interpolation, s is the arc length along the boundary, λ_K and $N_K(s)$ are the Lagrange multiplier and shape function at node K on the Dirichlet boundary respectively.

In EFG method, the source/sink terms are also distributed to the neighbouring nodes in the domain of influence of point under consideration and it is represented as

$$Q_w = \sum_{I=1}^{n} Q_w \, \phi_I(x,y) \,, \qquad q = \sum_{I=1}^{n} q \, \phi_I(x,y), \tag{10}$$

Therefore, using the trial solution, and Eqs. (9) and (10) in Eq. (8) gives

$$\begin{aligned} \int_\Omega \left(T_x \frac{\partial \phi_I(x,y)}{\partial x} h_I^{t+\Delta t} \right) \frac{\partial \phi_J(x,y)}{\partial x} \delta h_J \, d\Omega + \int_\Omega \left(T_y \frac{\partial \phi_I(x,y)}{\partial y} h_I^{t+\Delta t} \right) \frac{\partial \phi_J(x,y)}{\partial y} \delta h_J \, d\Omega \\ + \int_\Omega S \left(\phi_I(x,y) \frac{\partial h_I}{\partial t} \right) \phi_J(x,y) \, \delta h_J \, d\Omega \\ + \int_{\Gamma_S} \delta (N_K \lambda_K)^{\mathrm{T}} \left(\phi_I(x,y) h_I^{t+\Delta t} - h_D \right) d\Gamma + \int_{\Gamma_S} (N_K \lambda_K) \phi_J(x,y) \delta h_J \, d\Gamma \\ = q_B \int_{\Gamma_E} \phi_J(x,y) \delta h_J \, d\Omega - \int_\Omega (Q_w - q) \phi_I(x,y) \phi_J(x,y) \, \delta h_J \, d\Omega \end{aligned} \tag{11}$$

where $I, J = 1,2,3, \ldots \ldots \ldots n$.

The Eq. (11) can also be written as

$$\begin{aligned} \int_\Omega \left(T_x \frac{\partial \phi_I(x,y)}{\partial x} \frac{\partial \phi_J(x,y)}{\partial x} h_I^{t+\Delta t} \right) \delta h_J \, d\Omega + \int_\Omega \left(T_y \frac{\partial \phi_I(x,y)}{\partial y} \frac{\partial \phi_J(x,y)}{\partial y} h_I^{t+\Delta t} \right) \delta h_J \, d\Omega \\ + \int_\Omega \left(S \frac{\partial h_I}{\partial t} \phi_I(x,y) \phi_J(x,y) \right) \delta h_J \, d\Omega \\ + \int_{\Gamma_S} \delta {\lambda_K}^{\mathrm{T}} \left({N_K}^{\mathrm{T}} \phi_I(x,y) h_I^{t+\Delta t} - {N_K}^{\mathrm{T}} h_D \right) d\Gamma + \int_{\Gamma_s} (N_K \lambda_K) \phi_J(x,y) \delta h_J \, d\Gamma \\ = q_B \int_{\Gamma_E} \phi_J(x,y) \delta h_J \, d\Omega - \int_\Omega (Q_w - q) \phi_I(x,y) \phi_J(x,y) \, \delta h_J \, d\Omega \end{aligned} \tag{12}$$

The above integral form considers local numbering system for all the nodes in the local domain of influence. On transforming it from local to global in Eq. (12), it gives the following matrix form

$$\left[\left([K^{(1)}] + [K^{(2)}] \right) \{ h_I^{t+\Delta t} \} + [P] \left\{ \frac{\partial h_I}{\partial t} \right\} + [G]\{\Lambda\} - \{f\} \right] \delta H^{\mathrm{T}} + \left[[G^{\mathrm{T}}] \{ h_I^{t+\Delta t} \} - \{V\} \right] \delta \Lambda^{\mathrm{T}} \tag{13}$$

where

$$\{H^T\} = [h_1 \;\; h_2 \;\; h_3 \;\cdots\cdots\; h_N] \,, \tag{14a}$$

$$\{\Lambda^T\} = [\lambda_1 \;\; \lambda_2 \; \lambda_3 \;\cdots\cdots\cdots\; \lambda_{nt}] \tag{14b}$$

where N is the total number of nodes in the flow domain and nt is the total number of field nodes on the Dirichlet boundaries. The column vector $\{h_I^{t+\Delta t}\}$ gives the unknown nodal head values at time $t + \Delta t$. The elements of the global matrices $[K^{(1)}]$, $[K^{(2)}]$, $[P]$ and $[G]$ are computed as

$$K_{IJ}^{(1)} = \int_\Omega \left(T_x \frac{\partial \phi_I(x,y)}{\partial x} \frac{\partial \phi_J(x,y)}{\partial x} \right) d\Omega \,, \qquad K_{IJ}^{(2)} = \int_\Omega \left(T_y \frac{\partial \phi_I(x,y)}{\partial y} \frac{\partial \phi_J(x,y)}{\partial y} \right) d\Omega \tag{15a}$$

$$P_{IJ} = \int_\Omega S \, \phi_I(x,y) \phi_J(x,y) d\Omega \;\;, \qquad G_{KJ} = \int_{\Gamma_S} N_K \phi_J(x,y) d\Gamma \tag{15b}$$

Also, the elements of global vectors $\{f\}$ and $\{V\}$ are computed as

$$f_J = q_B \int_{\Gamma_E} \phi_J(x,y)d\Omega - \int_{\Omega} (Q_w - q)\phi_I(x,y)\phi_J(x,y)\,d\Omega \ , \quad V_K = \int_{\Gamma_S} N_K{}^{\mathrm{T}} h_D d\Gamma \tag{15c}$$

where $I,J = 1,2,3,\dots\dots\dots N$, $K = 1,2,3,\dots nt$ and $Q_w = q_w/a$, q_w is the pumping or recharge rate (m^3/d) and a is the nodal area (m^2). Since both δH and $\delta\Lambda$ are the arbitrary values, the Eq. (13) can be satisfied only if

$$\left([K^{(1)}] + [K^{(2)}]\right)\{h_I^{t+\Delta t}\} + [P]\left\{\frac{\partial h_I}{\partial t}\right\} + [G]\{\Lambda\} - \{f\} = 0 \tag{16}$$

$$[G^{\mathrm{T}}]\{H\} - \{V\} = 0 \tag{17}$$

The time derivative term in Eq. (16) is discretized using forward finite difference discretization (Pinder and Gray 1977; Wang and Anderson 1982) and it is expressed as

$$\frac{\partial h_I}{\partial t} = \frac{h_I^{t+\Delta t} - h_I^t}{\Delta t} \tag{18}$$

Both the Eqs. (16) and (17) can be collectively written in the following matrix form

$$\begin{bmatrix} K^* & G \\ G^{\mathrm{T}} & 0 \end{bmatrix} \begin{Bmatrix} H^* \\ \Lambda \end{Bmatrix} = \begin{Bmatrix} F \\ V \end{Bmatrix} \tag{19}$$

where

$$K_{IJ}^* = K_{IJ}^{(1)} + K_{IJ}^{(2)} + P_{IJ}\frac{1}{\Delta t} \ , \qquad H_I^* = h_I^{t+\Delta t}, \qquad F_I = f_I + P_{IJ} h_J^t \frac{1}{\Delta t} \tag{20}$$

The Eq. (19) represents the resultant system of equations obtained from the EFG method on utilizing the Lagrange multiplier method. However, the number of unknowns are increased by the size of $\{\Lambda\}$. On solving, Eq. (19) gives the nodal head values in the whole problem domain at time $t + \Delta t$.

3. NUMERICAL PROBLEM AND RESULTS

In the present study a two dimensional confined aquifer problem (Sharief 2007) consisting of three different zones (Figure 1) is selected. The aquifer area 1800 m × 1000 m with storativity = 0.004 has Dirichlet boundaries on the west (h = 100 m) and east (h = 95 m) sides whereas northern and southern boundaries are considered as impervious (no flow) boundaries. The transmissivity values for the three different zones in the x and y directions are T_x = 500 m^2/d and T_y = 300 m^2/d, T_x = 400 m^2/d and T_y = 250 m^2/d, and T_x = 250 m^2/d and T_y = 200 m^2/d respectively. The aquifer is recharged through the two aquitards lying between the northern nodes 6 to 24 and 42 to 60 with recharge rates 0.00024 and 0.00012 m/d respectively. A pond with seepage rate of 0.009 m/d is also located in the problem domain. The aquifer region consists of three pumping wells (PW) at nodes 26, 29 and 46 extracting groundwater at the rate of 200, 500 and 700 m^3/d respectively. It also contains a recharge well (RW) located at node 17 with injection rate of 800 m^3/d. Three observation wells are located in aquifer system at the nodes 21, 34 and 51.

In the simulation of groundwater flow by EFG method the aquifer domain is discretized using 60 nodes with nodal spacing of 200 m in both the x and y directions. The total 45 (9×5) number of background cells are used in the whole domain for the global integration with each cell size = 200 m × 200 m. In each cell 4 × 4 gauss points are used for the numerical integration by

Gauss quadrature scheme. The steady state analysis is used for obtaining the initial head distribution in the flow domain as shown in Figure 2. For this analysis all pumping and recharge wells are non-operational which are active during the transient flow analysis. The time step size (Δt) of 1 day is chosen for the system simulation and the total simulation time is 10000 days.

The flow behaviour in this aquifer system has been studied by Sharief (2007) using FEM method (60 nodes and 90 elements) and the available results are used for the verification of the present model.

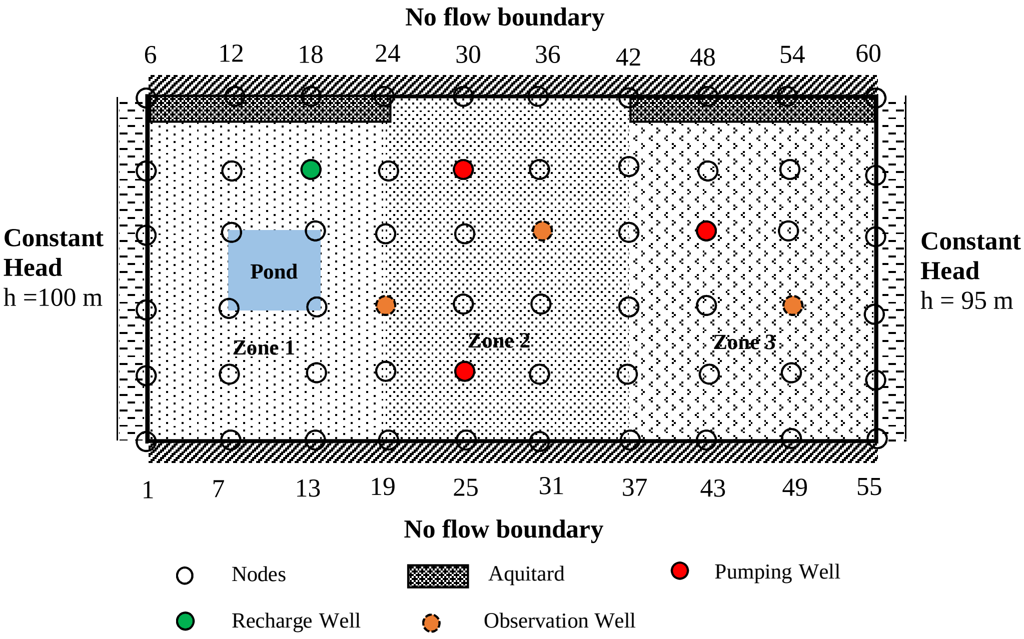

Figure 1. Meshless discretization of confined flow domain (Adapted from Sharief 2007).

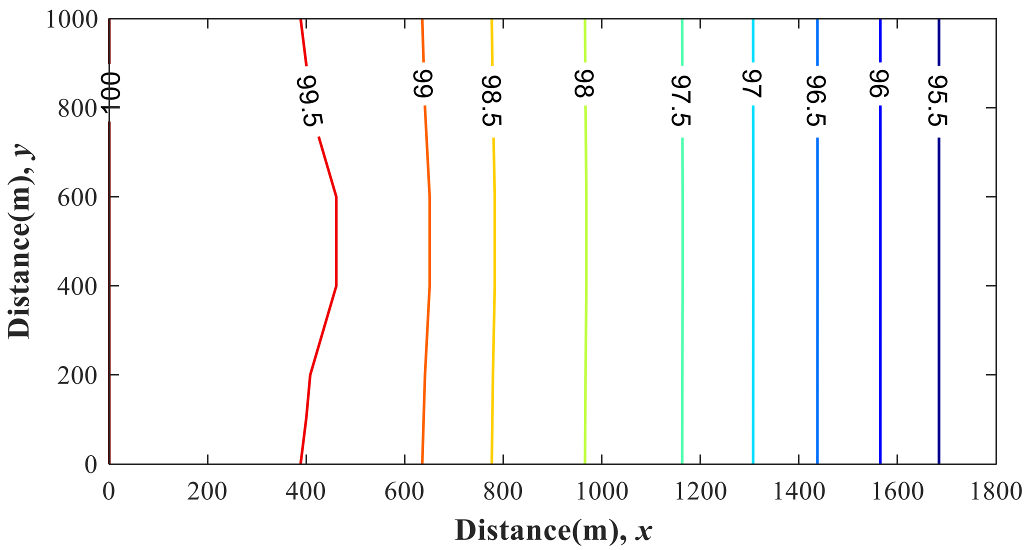

Figure 2. Steady state head distribution.

On fine tuning of the scaling parameter (Praveen Kumar and Dodagoudar 2009) within the range, d_{max}= 1~4 the value of 1.1 in EFG method gives the appropriate solution for this groundwater problem. The groundwater head distribution obtained from the EFG simulations at the end of the simulation period of 10000 days is shown in Figure 3. The comparison of head values at the observation wells after 10000 days by EFG simulations with the FEM solutions of Sharief are shown in Table 1. The EFG outputs are closely matching with the available FEM results. The effect of pumping and recharge wells at the observation wells in the flow region after 10000 days is also presented in Table 2. It is found that the head value increases when only the recharge well is active and it decreases when three pumping and the recharge well are active. However, it goes down significantly by 1.31 m due to pumping for observation well at node 34 as it is surrounded by all pumping wells.

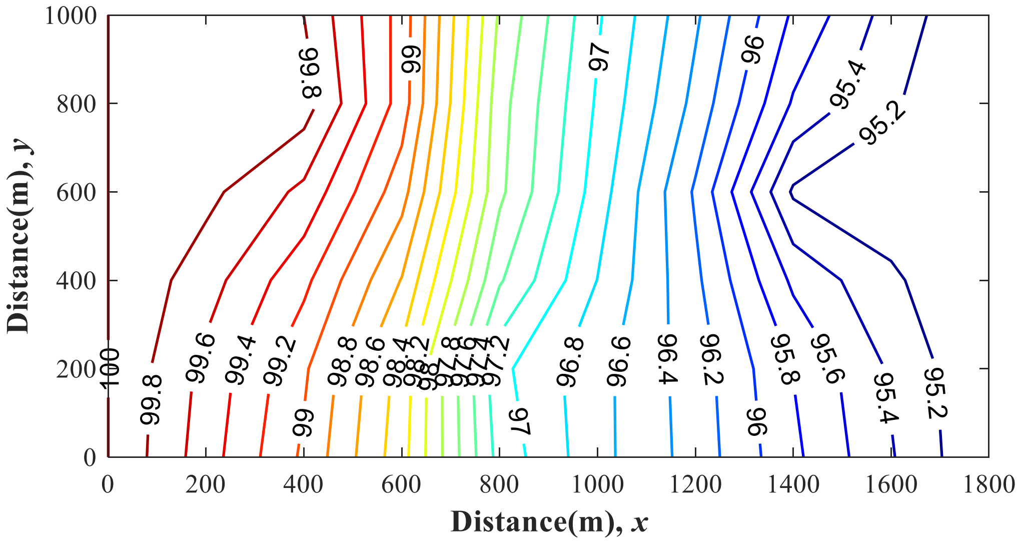

Figure 3. Head distribution in the flow region after 10000 days.

Table 1. Comparison of head distribution after 10000 days

Node No	Head in m (FEM)	Head in m (EFG)	Percentage Difference
21	98.20	98.59	0.396
34	97.10	96.90	0.206
51	95.25	95.23	0.021

Table 2. Effect of pumping and recharge wells at observation wells

Node No	Both PW and RW Inactive (m)	Only RW Active (m)	Both PW and RW Active (m)	Increase in Head due to RW (m)	Decrease in Head due to PW (m)
21	99.08	99.45	98.59	0.37	0.86
34	97.81	98.21	96.90	0.40	1.31
51	95.81	95.91	95.23	0.10	0.68

In this study an attempt is also made to run the EFG simulations with different time step sizes which are chosen as 5, 10, 20, 40 and 80 days respectively. The study found that the head values at three observation wells for Δt = 1 day were same as for sixteen fold increase in time step size. This suggests the robustness and stability of the EFG scheme.

4. CONCLUSIONS

In the present study an EFG discretization of a two dimensional groundwater flow model is presented. A MATLAB code is developed for the EFG model. The EFG shape functions derived from the MLS approximation do not obey Kronecker delta property. Thus, the Dirichlet boundary conditions are incorporated accurately using the Lagrange multiplier method. The elimination of the mesh generation procedure in pre-processing in this meshless method saves computational cost and time. However, the increased size of the resultant matrix in EFG method using Lagrange multipliers makes it computationally expensive. The EFG model is applied to a known problem (Sharief 2007) and its outputs are verified with his FEM results. The percentage difference between the groundwater head values predicted by the EFG and FEM simulations is less than 0.4 %. The EFG technique was found to be a stable numerical scheme as it gave the same head values for larger time step sizes. Therefore the present study suggests the application potential of meshless EFG method in simulating real field aquifers which are important for groundwater management.

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