

Annual sediment yield prediction by means of three soil erosion models at the basin scale

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Abstract: The objective of this study is the estimate of annual sediment yields deriving from soil and streambed erosion, at the outlet of a basin. Continuous simulations of soil and streambed erosion processes are performed in Nestos River basin, on an hourly time basis, for a five-year period. Three different soil erosion models are applied, as part of three Composite Mathematical Models. The first and third Composite Mathematical Models consist of three submodels: a rainfall-runoff submodel, a soil erosion submodel and a sediment transport submodel for streams, whilst the second one does not include a rainfall-runoff submodel and only consists of the soil erosion submodel and of several empirical sediment delivery ratio methods, for the estimation of the final sediment yield at the basin outlet. The rainfall-runoff submodel that is used for the computation of the surface runoff and the streamflow in the sub-basins, is the hydrologic model HEC-HMS 4.2. The first soil erosion submodel, utilized for the estimate of soil erosion in a sub-basin, is based on the relationships of Poesen (1985), the second is the widely known Universal Soil Loss Equation (USLE), while the third is the Modified version of USLE (MUSLE). The sediment routing for the first and third Composite Mathematical Models is achieved by means of the stream sediment transport model of Yang and Stall (1976). The three models are compared as to their ability to realistically predict annual sediment yields at the basin scale.

Key words: annual sediment yields, soil erosion, sediment transport, USLE, MUSLE

1. INTRODUCTION

The impacts of sediment yield in the form of concentrated sediment at the inlet of receptors, such as lakes, coastal zones, dams etc., is mainly limited in the receptor's morphological changes (sedimentation in lakes and dams, decline of coastal zones) and their potential contamination in the case that sediments are carriers of polluting factors. Hence, a more comprehensive evaluation results by assessing sediment yield in its preceding forms of soil erosion and sediment discharge. According to Collins et al. (2001), reliable information on soil erosion rates is an essential prerequisite for the design of targeted erosion and sediment control strategies. Despite the fact that sediments constitute the foundation stone for the balance and survival of - not only - the various aquatic and riparian ecosystems, but the very riverine systems themselves, they are - in the same time - negatively associated with a variety of complications. Erosion and deposition chisel and reshape the wet cross-sectional area, raising the risk of flooding in the case of excessive deposition. Soil erosion is directly connected to a series of environmental issues such as problems with the vegetation growth, increase of soil acidity levels, muddy floods etc. Excessive sedimentation is also associated to an increase of turbidity in streams, increase of water temperatures, decline of the dissolved oxygen's levels and destruction of the natural habitats of the aquatic and riparian life. Furthermore, sediments are potential carriers of contaminant factors. This is more evident in agricultural basins, where fertilizers and pesticides are in use.

The development of models that can reliably simulate soil erosion and sediment transport and predict sediment yields at a basin's outlet is indispensable. A large number of complex methodologies has been introduced during the last century, for predicting stream sediment discharge and sediment yield, none of which, however, enjoys a broad acceptance so far. Thus, the estimation of sediment discharge endures as one of the utmost elusive goals in the field of sediment

dynamics. This study presents the application of three different soil erosion models as a part of three Composite Mathematical Models (hereafter referred to as CMMs), aiming at the calculation of annual sediment yields at the outlet of Nestos River basin.

2. DESCRIPTION OF THE STUDY AREA

The under study part of Nestos River basin concerns the Greek mountainous part downstream of Platanovrysi dam (Figure 1). The basin extends to an area of 838.63 km², the altitude varies between 38 m and 1747 m, the average land slope of the basin is 37% and the length of the part of Nestos River that runs the basin, is approximately 63 km.

The characteristic of the basin is the presence of a dam at its upper boundary, which greatly affects the discharge, as well as the sediment transport in Nestos River. Along with this, two irrigation canals slightly upstream of the basin outlet (Egnatia bridge of Nestos River) are culpable of a recession of water, during the irrigation period, disrupting the final discharge at the basin outlet.

The four main soil types of the basin are: sandy clay loam, silty loam, loamy sand and silty clay loam. The basin is covered in its greatest part by forested and bushy areas and secondarily by crops, while a small fraction corresponds to urban areas and areas with no significant vegetation.

There are four meteorological stations in the area of study: “Mesochori”, “Prasinada”, “Oraio” and “SWAT” (Figure 1). The distribution of the meteorological stations to areas of influence was achieved with Thiessen polygons (Thiessen, 1911) (Figure 1).

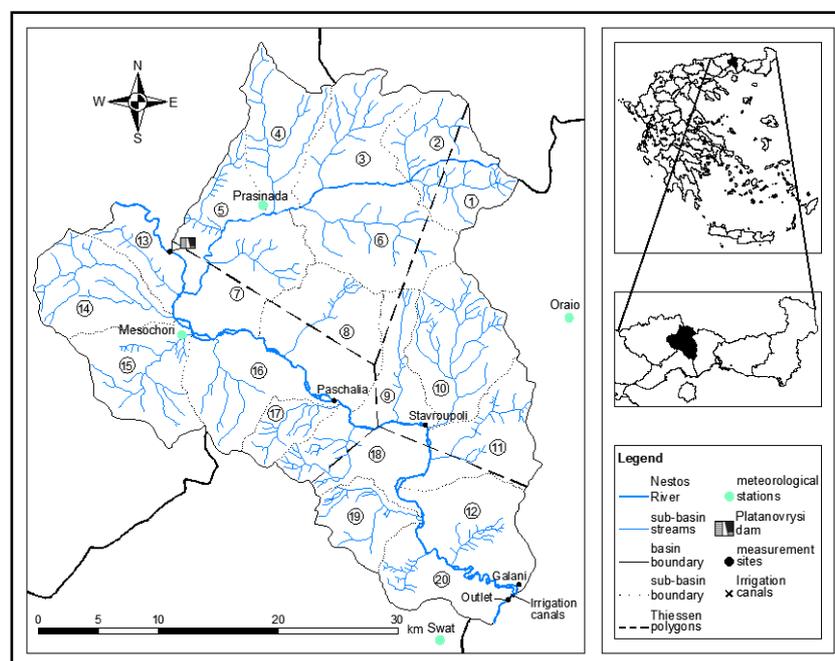


Figure 1. Nestos River basin

3. THEORETICAL DESCRIPTION OF THE MODELS

3.1 Rainfall-runoff submodel

A combination of known methods composes the rainfall-runoff submodel, which was built and simulated by the deterministic, semi-distributed hydrologic model HEC-HMS 4.2. More specifically, the Soil Conservation Service - Curve Number method (SCS, 1972) was used for the estimation of the hydrologic losses (mm) into the ground, as well as the amount of rainfall excess

(mm), the FAO-56 Penman-Monteith method (Allen et al., 1998) was used for the estimation of hydrologic losses due to evapotranspiration, whilst the transformation of rainfall excess to runoff hydrograph was achieved by means of the Soil Conservation Service dimensionless synthetic unit hydrograph (SCS, 1971). The Pasini's formula (Pasini, 1914) was applied for the estimation of the concentration time, while lag time was calculated by means of the - most commonly used - empirical equation of Soil Conservation Service (SCS, 1972). The model also comprises an exponential recession model to represent the time variation of baseflow (Barnes, 1939). Finally, the routing of the total discharge (direct runoff + baseflow) hydrograph from the outlet of a sub-basin to the outlet of the whole basin is enabled by means of the Muskingum-Cunge model, which is based on the well known hydrologic routing model of Muskingum (Cunge, 1969).

A much more thorough outline of the rainfall-runoff submodel can be found in Kaffas and Hrissanthou (2014) and Kaffas and Hrissanthou (2017).

3.2 Soil erosion submodels

3.2.1 First soil erosion submodel - Relationships of Poesen (1985)

The amount of soil loss in a basin is due to soil erosion caused by rainfall (detachment by raindrop impact) and soil erosion caused by runoff (shearing force of flowing water).

The estimation of soil erosion due to rainfall was achieved by the relationships of Poesen (1985):

$$q_{rs} = C(KE)r_s^{-1} \cos a \quad (1)$$

$$q_r = q_{rs} \left[0.301 \sin a + 0.019 D_{50}^{-0.22} \left(1 - e^{2.42 \sin a} \right) \right] \quad (2)$$

where q_{rs} is the mass of detached particles per unit area (kg/m^2); C is the soil cover factor; KE is the rainfall kinetic energy (J/m^2); r_s is the soil resistance to drop detachment (J/kg); a is the slope gradient ($^\circ$); q_r is the downslope splash transport per unit width (kg/m); and D_{50} is the median particle diameter (m).

The soil erosion due to runoff is calculated by means of Nielsen's equation (Nielsen et al., 1986), whilst the sediment transport capacity by runoff is computed by the modified formula of Engelund and Hansen (1967). The sediment yield that reaches the stream, is calculated by means of a comparison between the available sediment and the sediment transport capacity by runoff. A detailed version of the first soil erosion submodel can be found in Kaffas and Hrissanthou (2015).

3.2.2 Second soil erosion submodel - Universal Soil Loss Equation

The widely known Universal Soil Loss Equation (USLE) (Wischmeier and Smith, 1965) quantifies soil erosion caused by rainfall as a function of six factors, according to Equation (3):

$$A = R_{rainfall} \cdot K \cdot L \cdot S \cdot C \cdot P \quad (3)$$

where A is the soil loss per unit area [$\text{t}/(\text{ha yr})$]; $R_{rainfall}$ is the rainfall erosivity factor [$\text{MJ mm}/(\text{ha yr})$]; K is the soil erodibility factor [$\text{t h}/(\text{MJ mm})$]; L is the slope length factor; S is the slope steepness factor; C is the cover and management factor; and P is the support practice factor.

The calculation of the rainfall erosivity factor, $R_{rainfall}$, requires the intensity and duration of erosive rainfalls (İvrem et al., 2007). This would result to the derivation of individual unique values of $R_{rainfall}$ factor for each rainfall event. At this point, it has to be stated that the rainfall erosivity

factor, alone, anchors the USLE at the estimation of soil erosion by single rainfall events, making it an event-based erosion model. This study, however, aims at the estimation of annual sediment yields at the basin outlet. For this reason, an average annual value of rainfall erosivity factor was estimated by means of six empirical models, using mean monthly and annual rainfall depths.

Arnoldus (1977) modified Fournier's index to:

$$F = \sum_{i=1}^{12} \frac{p_i^2}{P} \quad (4)$$

where F is the modified index value; p_i is the average monthly rainfall depth (mm); and P is the average annual rainfall depth (mm).

The formulas of the six empirical models, used for the estimation of $R_{rainfall}$, are given in Equations (5) - (10), below:

Arnoldus (1977):

$$R_{rainfall} = 0.302 \cdot F^{1.93} \quad (5)$$

Arnoldus (1980):

$$R_{rainfall} = [(4.17 \cdot F) - 152] \cdot 17.02 \quad (6)$$

Lo et al. (1985):

$$R_{rainfall} = 38.46 + 3.48 \cdot P \quad (7)$$

Renard and Freimund (1994):

$$R_{rainfall} = 0.739 \cdot F^{1.847} \quad (8)$$

Renard and Freimund (1994):

$$R_{rainfall} = 0.0483 \cdot P^{1.61} \quad (9)$$

Yu and Rosewell (1996):

$$R_{rainfall} = 3.82 \cdot F^{1.41} \quad (10)$$

Williams (1995) proposed the following equation for the soil erodibility factor:

$$K = f_{csand} \cdot f_{cl-si} \cdot f_{orgc} \cdot f_{hisand} \quad (11)$$

where f_{csand} is a factor that reduces the K in soils with high coarse-sand contents and increases it in soils with little sand; f_{cl-si} is a factor that reduces the K in soils with high clay to silt ratios; f_{orgc} is a factor that reduces the K in soils with high organic carbon content; and f_{hisand} is a factor that reduces the K in soils with extremely high sand contents.

The cover and management factor, C , is estimated as a cover factor, due to the fact that the basin is mountainous and the lack of management practices, as a function of the various cover types

of the basin.

The slope length factor, L , and the slope steepness factor, S , are calculated as a combined topographic factor, LS , by Equation (12):

$$LS = \left(\frac{L_s}{22.1} \right)^m \cdot [65.41 \cdot \sin^2(a) + 4.56 \cdot \sin a + 0.065] \quad (12)$$

$$m = 0.6 \cdot [1 - \exp(-35.835 \cdot s)] \quad (13)$$

where L_s is the slope length (m); m is an exponential term; a is the angle of the slope; and s is the average slope of the sub-basin (m/m).

The support practice factor, P , expresses the impact of support and conservation practices on the average annual soil loss. The closer to unity the P factor, the less the support practices and conservations plans. An average value of 0.9502 was considered, for the area of study (Panagos et al., 2015).

3.2.3 Third soil erosion submodel - Modified Universal Soil Loss Equation

The key difference between the Universal Soil Loss Equation (USLE) and its Modified version (MUSLE) (Williams, 1975) is that MUSLE, in contrast to USLE, estimates sediment yield due to runoff erosion instead of rainfall erosion. Thus, the rainfall erosivity factor, $R_{rainfall}$, in USLE, is replaced in MUSLE by a runoff erosivity factor, R_{runoff} . The R_{runoff} factor allows MUSLE to be applied for both event and continuous (long-term) erosion modeling, in contrast to the $R_{rainfall}$ factor which anchors USLE to the estimation of soil loss on an event basis. However, rainfall is also taken into account in MUSLE, in the sense that the runoff erosivity factor is a function of antecedent moisture condition, as well as of rainfall energy. Another critical difference between USLE and MUSLE that needs to be taken under consideration, in relevant studies, is that the USLE estimates the amount of eroded soil per unit area and time, whilst the MUSLE estimates the sediment yield - due to soil erosion - that reaches the main streams per unit of time. The rest of the MUSLE factors (K, L, S, C, P) are computed in the same way as in the USLE.

The Modified Universal Soil Loss Equation (MUSLE) is given below:

$$sed = 11.8 \cdot (Q_{surf} \cdot q_{peak} \cdot area_{sub})^{0.56} \cdot K \cdot L \cdot S \cdot C \cdot P \quad (14)$$

where sed is the sediment yield, in t, per unit of time; Q_{surf} is the surface runoff volume (mm/km²); q_{peak} is the peak runoff rate (m³/s); $area_{sub}$ is the area of the sub-basin (km²).

3.3 Models for the calculation of the final sediment yield at the basin outlet

3.3.1 Stream sediment transport model of Yang and Stall (1976)

Sediment transport capacity by streamflow is estimated from the sediment concentration in the stream, which is computed by the relationships of Yang and Stall (1976):

$$\log c_{ts} = 5.435 - 0.286 \log \frac{wD_{50}}{v} - 0.457 \log \frac{u_*}{w} + \left(1.799 - 0.409 \log \frac{wD_{50}}{v} - 0.314 \log \frac{u_*}{w} \right) \log \left(\frac{us}{w} - \frac{u_{cr}S}{w} \right) \quad (15)$$

where c_{ts} is the total sediment (sand) concentration by weight (ppm); w is the terminal fall velocity of sediment particles (m/s); D_{50} is the median particle diameter (m); ν is the kinematic viscosity of the water (m²/s); s is the energy slope; u is the mean flow velocity (m/s); u_{cr} is the critical mean flow velocity (m/s); and u_* is the shear velocity (m/s).

A more thorough outline of the stream sediment transport submodel can be found in Kaffas and Hrisanthou (2015).

3.3.2 Sediment delivery ratio

As it has already been mentioned, the USLE estimates the amount of eroded soil per unit area and time. This indicates that a method for the estimation of the amount of eroded soil that gets deposited in the basin, and does not reach the main stream, has to be included. Four different empirical models estimating the sediment delivery ratio - namely the percentage of the total erosion amount that reaches the outlet of the basin - were used in this study. However, as with all empirical models, these formulas have been developed for certain conditions and limitations and they do not guarantee efficient application in an environment that differs from the one they were primarily created for. The following formulas [Equations (16) - (19)] were developed for certain sites in the USA.

Maner (1958):

$$\log(DR) = 2.94259 - 0.82362 \cdot \log\left(\frac{\ell}{RL}\right) \quad (16)$$

where DR is the delivery ratio (%); ℓ is the length of the main stream (ft); and RL is the difference between the altitude of the two ends of the main stream (ft).

Maner (1962):

$$\log(DR) = 1.8768 - 0.14191 \cdot \log(10 \cdot FL) \quad (17)$$

where DR is the delivery ratio (%); and FL is the basin area (mi²).

Williams and Berndt (1972):

$$DR = 0.627 \cdot s^{0.403} \quad (18)$$

where DR is the delivery ratio (decimal); and s is the mean bed slope of the main stream.

Williams (1977):

$$DR = 1.366 \cdot 10^{-11} \cdot FL^{-0.0998} \cdot \left(\frac{RL}{\ell}\right)^{0.3629} \cdot CN^{5.444} \quad (19)$$

where DR is the delivery ratio (decimal); FL is the basin area (km²); RL is the difference between the altitude of the two ends of the main stream (m); ℓ is the length of the main stream (km); and CN is the average value of the Curve Number for the entire basin.

The aforementioned models were applied to the total erosion amount (soil erosion and streambed erosion). The USLE estimates an overall mean annual amount of soil erosion, and the soil erosion products cannot be routed through a stream sediment transport model, as the one presented in Subsection 3.3.1. Hence, the erosion of the streambed and banks was estimated empirically as a fraction (20%) of soil erosion by bibliographic suggestions (Roehl, 1962).

4. APPLICATION OF THE MODELS

HEC-HMS ran continuously for a five year period, from 2009 to 2013, resulting in runoff, baseflow and total stream discharges (surface runoff + baseflow) at an hourly time step. The rainfall depths and runoff hydrographs constitute the decisive input data to the first soil erosion submodel (Poesen), whilst only the runoff hydrographs were used as input to the third soil erosion submodel (MUSLE). The outcome of the two models is the sediment yield that reaches the main streams of the sub-basins. All the simulations were carried out at the sub-basin scale and for this reason the basin was divided into twenty natural sub-basins (Figure 1). The application of the first and third CMMs results to hourly sediment yields, which are summed to provide the annual sediment yields at the basin outlet.

Rainfall data for an eleven-year period, from 2003 to 2013, were used to define the $R_{rainfall}$ factor, which was calculated separately for each of the twenty sub-basins. Each of the Equations (5) - (10) provided a different value for $R_{rainfall}$ factor and, finally, an average value was obtained for each sub-basin. The $R_{rainfall}$ factor varies between 1413.37 MJ mm/(ha h yr) and 2291.96 MJ mm/(ha h yr). The result from the USLE is the total soil erosion amounts of the sub-basins which were summed to provide the amount of soil erosion for the entire basin. This was multiplied by 1.2 to reckon in the streambed and bank erosion. Finally, the sediment delivery ratio, DR , was computed by each of the Equation (16) - (19) which provided an average value of 10.24%. This means that 10.24% of the total erosion amount (soil erosion + erosion from the main streams) reaches the outlet of the basin.

5. RESULTS

Table 1 displays the annual sediment yield values as they resulted from the application of the first and third CMMs. The sediment yields are provided in t/yr, while these values are divided by the area of the basin to provide sediment yields in t/km²/yr.

Table 1. Annual sediment yields at the basin outlet

Year	Annual sediment yield (10 ⁶ t/yr)		Annual sediment yield (t/km ² /yr)	
	First Composite Mathematical Model (Poesen)	Third Composite Mathematical Model (MUSLE)	First Composite Mathematical Model (Poesen)	Third Composite Mathematical Model (MUSLE)
	2009	1.05	1.23	1250
2010	1.09	1.35	1303	1611
2011	0.41	0.62	482	734
2012	1.31	1.12	1567	1334
2013	0.98	1.10	1166	1311

In Table 2, the average annual sediment yield values of all the three CMMs are given for the period 2009-2013, in t/yr and t/km²/yr.

Table 2. Average annual sediment yields at the basin outlet for the period 2009-2013

Model	Average annual sediment yield (10 ⁶ t/yr) for the time period 2009-2013	Average annual sediment yield (t/km ² /yr) for the time period 2009-2013
First Composite Mathematical Model (Poesen)	0.97	1154
Second Composite Mathematical Model (USLE)	1.58	1888
Third Composite Mathematical Model (MUSLE)	1.08	1292

6. CONCLUSIONS

It is concluded that there is a very good approximation between the results of the first (Poesen) and the third (MUSLE) CMMs. The sediment yield values of the second CMM (USLE) are also high to those of the other two models, but slightly higher. This can be attributed to the various empirical methods included in the second CMM, as well as to the fact that the annual sediment

yields in the first and third CMMs resulted as a sum of hourly sediment yields. This high level of disaggregation (hourly time steps) imparts to these models a very high degree of precision. It could be stated that the connection of the submodels, in the CMM, through the input-output data ensures the almost parallel running of the models, which approximates the natural reality.

Knowledge of the sediment yield at the outlet of a basin or at any other point of interest, as well as the study of its preceding processes (soil and streambed erosion) constitutes the most vital information when it comes to the hydromorphological study at the basin scale and the study of riverine systems.

Given the fact that the process of measuring sediment yields is a difficult and laborious task, yet of great significance for a variety of reasons, the CMMs presented here, can be used in both the Greek and the Mediterranean mountainous terrain to successfully estimate soil erosion, sediment discharges and sediment yields at the basin scale.

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