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Spectral analysis and topological and energy metrics for water network partitioning of Skiathos island

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Abstract: Water network partitioning (WNP) into supply clusters aims at improving management of the whole system and subsequent definition of District Meter Areas (DMAs), simplifying water budget and pressure management as well as the identification and reduction of water losses. This solution is a strategic option used by water utilities to control and operate their Water Distribution Networks (WDNs). Thus, it is important to design DMAs in optimal way, inserting flow meters and gate valves in boundary pipes. This is a demanding task, since closure of some network links can worsen network performance in terms of hydraulic and topologic redundancy. This paper presents the application of a novel optimization approach, based on network theory and Graph Partitioning (GP), coupled with heuristic optimization techniques. The methodology uses SWANP software to cluster the network and proceeds with a novel approach for the partitioning phase, based on the minimization of a multi-objective function including both energy and spectral metrics. The methodology is tested on Skiathos Island, Greece WDN. We present interesting preliminary results based on the analysis of several performance indices that measure resilience and robustness of the partitioned network. Overall, an adequate solution to the optimal definition of hydraulic districts through DMAs is presented.

Key words: water network partitioning, spectrum, topological metrics, optimization, multi-objective function

1. INTRODUCTION

The paradigm of "divide and conquer" applies fruitfully on clustering data, objects or other sets into smaller subsets. Clustering allows identifying sub-groups in a way that objects within are closely related (Hastie et al., 2001), showing similarity expressed in different ways according to the purpose of the problem (Herrera et al., 2012). The division in groups allows management improvement and analysis simplification. In this regard, the application of "divide and conquer" to WDNs, by the insertion of gate valves and flow meters, allows improvements in the WDN management, such as water balance (AWWA, 2003) and pressure control (Alonso et al., 2000), facilitating detection and reduction of water losses (Xia and Guo-Jin, 2010) and obstructing accidental or intentional water contamination (Grayman et al., 2009; Di Nardo et al., 2015).

Although this strategy is generally used to improve WDNs operation, WNP in permanent DMAs, achieved by closure of some pipes by means of gate valves, reduces topologic and energy redundancies (Mays, 2000). Additionally, this may significantly decrease hydraulic performance and network robustness (Di Nardo et al., 2013). Traditionally, defining DMAs has been strongly empirical without a proper methodology. In the last decades, studies that have allowed to overcome empirical approaches (review by Perelman et al., 2015) generally suggest two main phases: a) *clustering*, for the network subsets shaping and dimensioning through minimization of edge-cuts and balance of nodes number for each district, by means of graph theory algorithms (Tzatchkov et al., 2006; Alvisi and Franchini, 2014), spectral approach (Herrera et al., 2010; Di Nardo et al., 2017), multi-agent approach (Izquierdo et al., 2011), community structure (Diao et al., 2013, Di

Nardo et al., 2015); and b) *dividing*, for the selection of pipes where the insertion of flow meters or gate valves is needed to minimize both the economic investment and the hydraulic performance deterioration, based on iterative (Ferrari et al., 2014; Diao et al., 2013) or Genetic Algorithms (GA) (Di Nardo et al., 2016). To achieve efficient network division, geometrical, topological and hydraulic characteristics need to be considered, guaranteeing correct layout and function of hydraulic districts, as well as satisfying minimum service level at all demand nodes.

This paper aims to contribute to the arduous problem of WDN partitioning by proposing a technique that efficiently and automatically defines optimal DMAs, based on the minimization of a novel multi-objective function that considers simultaneously both topological and hydraulic aspects. In particular two basic metrics from Complex Networks Theory (Boccaletti et al., 2006) were selected: a) average path length L (Watts and Strogatz, 1998), fundamental for the characterization of different network models, and b) algebraic connectivity λ_2 (Fiedler, 1973) on which spectral theory is mainly based (Von Luxburg, 2007). After the first clustering phase, in which the sub-regions are defined through a GP method, the selection of pipes in which gate-valves or flow meters have to be alternatively inserted is obtained by minimizing hydraulic deterioration, while preserving communication (low value of average path length) and connectivity (high value of algebraic connectivity λ_2) through a heuristic optimization criterion.

The procedure is tested on the WDN of Skiathos Island, Greece, through the analysis of dimensional and robustness indices (Yazdani and Jeffrey, 2010) that measure the performance of the partitioned network.

2. METHODOLOGY

The partitioning procedure used consists of: Phase 1, Water network clustering and Phase 2, Water network physical dividing.

Regarding *Phase 1*, considering a simple graph $G=(V,E)$, where V is the set of n vertices v_i (or nodes) and E is the set of m edges e_l (or links), a k-way graph clustering problem consists of partitioning V vertices of G into k subsets, $P_1, P_2, \ldots, P_k$ (Di Nardo et al., 2017).

Graph clustering can be achieved by optimizing an objective function related to one of the clustering aims (i.e. clusters balancing, minimization of boundary pipes) (Boccaletti et al., 2006; Fortunato, 2010). In this paper, for the clustering phase, a GP algorithm was implemented in SWANP 3.0 software (Di Nardo et al., 2013).

GP is a computer science technique developed to solve problems that require huge computational power, such as finite element based simulations that require the distribution of a finite element mesh among different processors. Therefore, GP techniques were developed in computer science for optimal allocation of a computational mesh in parallel or distributed computing architectures (Di Nardo et al., 2013).

Specifically, *Phase 1* was achieved with SWANP 3.0, through a Multi-Level Recursive Bisection (MLRB) algorithm, a highly effective method for the k-way graph clustering (Karypis and Kumar 1998a, b). MLRB techniques are based on the following principal phases. a) *Coarsening*, which simplifies the original graph $G_0=(V_0, E_0)$, through a node aggregation that generates a sequence of smaller graphs $G_i=(V_i, E_i)$, namely the coarser graphs, each with fewer vertices so that $|V_i|<|V_i-1|$. This phase is essential for reducing the dimensions of a graph and, consequently, the complexity of finding a k-way clustering. b) *Clustering*, through which a k-way clustering is found by recursive bisection, performing a sequence of 2-way clustering. In this phase, through an optimization procedure, a clustering is obtained with the minimum number of edges N_{ec} and with a balanced number of vertices n_p belonging to each subset, with the optimal limit condition of n_p=n/k. c) *Uncoarsening* (also with refinement), a projection from the coarser graph G_m back to the original graph G_0 by going through the graphs $G_{m-1}, G_{m-2}, \ldots, G_1$ (uncoarsening), with a local optimization of the partition (refinement) using heuristic algorithms (Hendrickson and Leland, 1993), thus decreasing the edge-cuts by moving a vertex from one partition to another. This was implemented in the SWANP 3.0 software (Di Nardo et al., 2013).

After defining the set N_{ec} of the edge-cuts, *Phase 2* aims at establishing the pipes in which, gate-valves N_{gv} and flow meters N_{fm}=(N_{ec}-N_{gv}) need to be inserted. This can be assimilated to a WDNs valve placement problem, an NP-hard problem (Bodlaender et al., 2010) that requires heuristic algorithms to find optimal solutions in the following space of combinations N_c computed by the following binomial coefficient:

$$N_C = \binom{N_{ec}}{N_{fm}} \tag{1}$$

To achieve the goal of *Phase 2*, a GA was designed (Di Nardo et al., 2013). In this paper, once the number of flow meters N_{fm} is fixed, to define the optimal location of gate valves and flow meters the following five Objective Functions (OFs) were tested, combining energy, topological, and spectral metrics:

$$OF_1 = \frac{P_N}{P_N^*} \tag{2}$$

where $P_N = \gamma \sum_{i=1}^{n} Q_i H_i$ is the total nodal power of the partitioned layout, γ is the water specific weight, Q_i and H_i are respectively the water demand and the hydraulic head at each node, while P^*_N is the total nodal power of the original network layout;

$$OF_2 = \frac{APL^*}{APL} \tag{3}$$

where $APL = \sum_{\forall s \neq t} \sigma(s,t)/0.5n(n-1)$ is the average path length (Watts and Strogatz, 1998) of the partitioned layout, n is the total number of the network vertex, $\sigma(s, t)$ is the shortest path between two nodes s and t, computed as the number of edges or the sum of the weights of the edges respectively for unweighted/weighted network, APL^* is the average path length of the original network layout;

$$OF_3 = \frac{\lambda_2}{\lambda_2^*} \tag{4}$$

where λ_2 is the second smallest eigen-value of Laplacian matrix (Fiedler, 1973) of the partitioned layout, defined as $L=D-A$, in which D=diag(d_i) is the diagonal matrix of the node-degrees, A=(a_{ij}) is the adjacency matrix of graph G in which the element $a_{ij} \neq 0$ if the nodes i and j are connected, for an unweighted network a_{ij}=1, while for weighted network a_{ij}=w_{ij} with w_{ij} equal to the weight associated to the link ij; λ^*_2 is the corresponding value of the original network layout;

$$OF_4 = \frac{P_N}{P_N^*} + \frac{APL^*}{APL} \tag{5}$$

$$OF_5 = \frac{P_N}{P_N^*} + \frac{\lambda_2}{\lambda_2^*} \tag{6}$$

The last two multi-objective functions consider the effect of hydraulic energy coupled with network topology (OF4) and spectral metric (OF5). This multi-objective optimization approach is known as the "weighted-sum" or "scalarization" method (Caramia and Dell'Olmo, 2008) that represents a new optimization problem with a unique objective function composed by two functions.

3. SIMULATION RESULTS

The proposed procedure was tested on the medium size WDN of Skiathos, a Greek touristic Aegean Sea island of 6,000 inhabitants. The network (Figure 1) was modeled in EPANET2 (Rossman, 2000), with m=282 pipes, n=184 nodes, one source and a total pipe length L_{tot}=18.25 km.

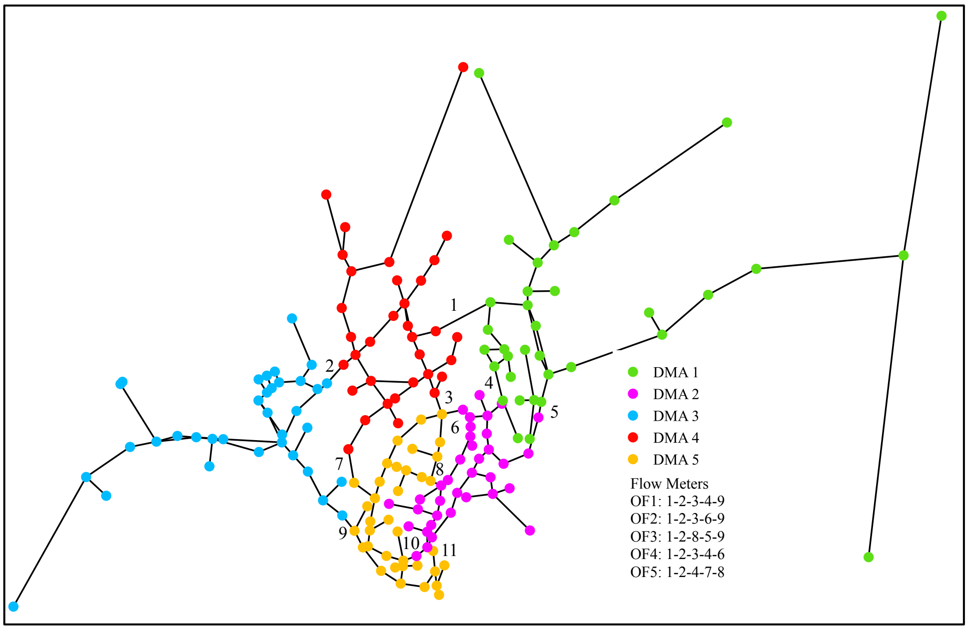

Figure 1. Skiathos WDN with representation of DMAs and boundary pipes.

Firstly, the WDN was clustered (*Phase 1*) by SWANP 3.0 software in 5 DMAs of approximately equal nodes as confirmed by the balancing index I_B=1.02 and with N_{ec}=11 boundary pipes (or edge-cuts). Then, starting from the same clustering layout, the number of flow meters was fixed to N_{fm}=5 (consequently, N_{gv}=6) and the dividing phase (*Phase 2*) was carried out for each previously defined OFs (equations 2, 3, 4, 5 and 6). For comparing simulation results, two categories of Performance Indices (PIs) were involved:

a) *resilience performance indices*: mean node pressure h_{MEAN}, maximum node pressure h_{MAX}, minimum node pressure h_{MIN}, standard node pressure deviation $h_{SD,}$ resilience index I_R (Todini, 2000) and resilience deviation index I_{RD};

b) *robustness performance indices*: index of robustness deviation, R_D, obtained as a modification of the Piraveenan et al (2013) robustness coefficient R below provided, which evaluates the network ability to avoid malfunctioning when a fraction of its elements, links or nodes, is damaged:

$$R = \frac{0.5 \cdot S_0 + \sum_{j=1}^{m-1} S_j + 0.5 \cdot S_m}{0.5 \cdot S_0^* + \sum_{j=1}^{m-1} S_j^* + 0.5 \cdot S_m^*} \quad (7)$$

where S_j represents the largest size of connected component of water partitioned network after the removal of j links and S^*_j the corresponding value for the original network. The largest size of connected component S will become S=1 only when all links are removed.

Values of OFs and energy and topological performance indices are reported, respectively, in Tables 1 and 2 for all WNP layouts obtained with different OFs (WNP$_{OF}$) and for the original network layout (ONL).

The impact of a multi-objective approach on the positioning of flow meters and gate valves in the boundary pipes can be observed in Table 1, which depicts different values when energy, topological or spectral metric, or their combinations, were used in the OF. As expected, P_N is higher with WNP$_{OF1}$ while the lowest increase of *APL* corresponds to the partitioned layout obtained with WNP$_{OF2}$ and the best λ_2 value is obtained with WNP$_{OF3}$. Nevertheless, the optimization of WNP$_{OF4}$ and WNP$_{OF5}$ provided interesting results. The values of P_N of WNP$_{OF4}$ and WNP$_{OF5}$ are substantially equal to the partitioned layout obtained with *OF1* (computed only with energy consideration) and better than the WNP carried out by optimization of WNP$_{OF2}$ and WNP$_{OF3}$ (computed without energy considerations). Furthermore, the solutions obtained with *OF4* and *OF5* improve significantly the values of *APL* and λ_2 of WNP$_{OF1}$, although they obviously worsen the corresponding values of WNP_{OF2} and WNP_{OF3} (computed only with topological and spectral considerations, respectively).

Table 1. Simulation results obtained with GA using different OFs vs ONL

	ONL	WNP_{OF1}	WNP_{OF2}	WNP_{OF3}	WNP_{OF4}	WNP_{OF5}
P_N (kW)	104.846	104.588	102.431	102.615	104.266	104.005
APL (m)	981.63	1135.72	1022.62	1098.03	1044.74	1097.22
λ_2	0.4815	0.1315	0.2646	0.3206	0.2279	0.2670
OF	-	0.998	0.960	0.666	1.934	1.546

Table 2. Resilience and robustness performance indices of all WNP layouts vs ONL

	ONL	WNP_{OF1}	WNP_{OF2}	WNP_{OF3}	WNP_{OF4}	WNP_{OF5}
h_{MIN} (m)	7.06	7.10	7.28	7.09	6.99	6.97
h_{MEAN} (m)	47.98	47.91	47.17	47.16	47.77	47.68
h_{MAX} (m)	61.54	61.54	61.54	61.54	61.54	61.54
h_{SD}(m)	10.29	10.30	10.27	10.35	10.30	10.32
I_R	0.961	0.957	0.921	0.924	0.951	0.947
I_{RD} (%)	-	0.42	4.16	3.85	1.04	1.46
R_D (%)	-	69.85	77.34	72.52	69.09	64.03

The analysis results in terms of performance indices (Table 2) show that maximum and mean nodal pressure of the original WDN, h_{MAX}=61.54 m and h_{MEAN}=47.98m, are both significantly higher than the design pressure of h^*=15 m required to satisfy water demand at all nodes. However, minimum pressure h_{MIN}=7.06 m is lower than h^*, which means that there are nodes (particularly only one node) for which the design pressure limit is not fulfilled. This also determines a high value of standard deviation h_{SD}=10.29 m. Moreover, the system shows high energy resilience I_R=0.961 and, consequently, "good availability" to be partitioned without a decrease in hydraulic performance (Greco et al, 2012).

All WNP layouts show satisfying energy performance with deviation of resilience index I_{RD}<5.00%. Indeed, values of minimum, mean and maximum pressure are practically equal to the value of the original network (Table 2) with best results for WNP_{OF1}, WNP_{OF4} and WNP_{OF5} with an I_{RD}<1.5%. In terms of robustness, instead, the best results are by far those associated to the network layouts WNP_{OF2} (R_D=77.34) and WNP_{OF3} (R_D=72.52). The simulation results confirm the effectiveness of the proposed optimization approach with significant and coherent changes of objective functions as reported in Table 1. Therefore, evidently, the choice of the best partitioning layout depends on the operator objectives. Anyway, for this case study, the layout WNP_{OF1} shows a good compromise between energy (I_{RD}=0.42) and robustness (R_D=69.85). Furthermore, although WNP_{OF4} and WNP_{OF5} layouts provide better values of *APL* and λ_2 metrics than WNP_{OF1} layout, the robustness deviation of WNP_{OF1} layout (R_D=69.85) is better.

In Figure 1, to show all layout solutions, the boundary pipes N_{ec} are highlighted using a numerated label and, for each WNP, the index of pipes where flow meters were inserted are reported.

4. CONCLUSIONS

The proposed multi-objective optimization approach based on energy, topological and spectral metrics represents an effective methodology to define the optimal positioning of flow meters and gate valves on boundary pipes. Despite the small size of the WDN tested, results show a remarkable sensibility of energy, topological and spectral metrics to describe alterations of the network.

Therefore, the proposed approach, which will be applied in future studies on larger WDNs, provide water utilities with a novel tool to define optimal water network partitioning by evaluating, simultaneously, network resilience and robustness.

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