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Incorporating uncertainty in the design of water distribution systems

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Abstract: During the design stage of urban water supply and distribution systems several assumptions and projections to future horizons are made. The analysis and design is then based on a deterministic basis in which all the involved parameters have crisp values. In reality however, most of the parameters exhibit significant variability over the life span of the system. The resistance to flow is among the most important parameters exhibiting variability. This is caused by the variability of the friction coefficient of pipes and the variability of internal diameters of the pipes. This study attempts to incorporate these uncertainties in the analysis and design of urban water supply and distribution systems, following a copula approach. Because of the low data availability, fuzzy membership functions are assigned to describe both variability of friction coefficients and variability of internal pipe diameters. Instead of the min intersection, the use of copulas is proposed in order to combine the uncertainties and calculate the fuzziness of the pipe resistance to flow. Thus, the input variables are fuzzy numbers and the problem is to find the output of the system which will consists also of fuzzy quantities. The system itself is described by the set of precise (crisp) nonlinear Q-equations. Finally, the uncertainty analysis concludes to an optimization problem. A numerical example is presented to demonstrate this innovative methodology.

Key words: Water distribution system, Uncertainties in design, Resistance to flow, Pipe roughness, Copulas, Fuzzy sets, Extension principle

1. INTRODUCTION

At the design stage of urban water supply and distribution systems several assumptions and projections to future horizons are made. All the values set on the involved parameters are considered constant throughout the operational life of the system. In reality, however, several parameters exhibit significant variability over time. The future water demand and the resistance to flow are among these important uncertain parameters. As known, the variability of the resistance to flow is caused by the changes of the friction coefficient and the internal diameters of the pipes during the life of the system.

This article presents a hybrid fuzzy approach for incorporating these uncertainties in the analysis and design of urban water supply and distribution systems.

2. BASIC NOTIONS

A fuzzy number is a fuzzy set satisfying the properties of convexity and normality. It is defined in the axis of real numbers and its membership function is a piecewise continuous function. For simplicity, a fuzzy number representing the uncertainty in both the pipe roughness and the internal pipe diameters, is a fuzzy triangular number.

An extension of a triangular fuzzy number is the LR fuzzy number with membership function of a non-linear shape (Dubois and Prade 1978). A fuzzy number is of the LR-type if there exist functions L (for left) and R (for right) with:

$$\begin{cases} \mu_A(x) = \begin{cases} L\left(\dfrac{x-\alpha_1}{\alpha_2-\alpha_1}\right) & \text{if } \alpha_1 \le x \le \alpha_2 \\ R\left(\dfrac{\alpha_3-x}{\alpha_3-\alpha_2}\right) & \text{if } \alpha_2 \le x \le \alpha_3 \\ 0 & \text{otherwise} \end{cases} & (1.a) \\ \text{where L and R are two non-decreasing shape functions which satisfy the following equations:} \\ L:[0,1]\to[0,1] \text{ and } R:[0,1]\to[0,1] & \\ L(0)=R(0)=0,\ L(1)=R(1)=1 & (1.b) \end{cases}$$

The α-cut set of the fuzzy number A (with $0 < \alpha \le 1$), is the key idea to move from the fuzzy to the crisp sets and it is defined as follows (Klir and Yuan 1995; Zimmermann 1991):

$$\tilde{A}_\alpha = \left\{x \middle| \mu_{\tilde{A}}(x) \ge \alpha,\ x \in \Re\right\}. \tag{2}$$

One can notice that the α-cut set is a crisp set determined from the fuzzy set according to a selected value of the membership function, and reciprocally, a fuzzy set can be derived from a significant number of α-cut sets.

The crisp set including all the elements with non-zero membership function is the 0-cut which is defined as follows (Buckley and Eslami 2002):

$$\tilde{A}_{0^+} = \left\{x \middle| \mu_A(x) > 0,\ x \in \Re\right\} \tag{3}$$

We can now extend the operation of the usual crisp functions, if the inputs are fuzzy sets, based on the extension principle which is briefly presented below.

Let X be a Cartesian product of universe $X = X_1 \times X_2 \times ... \times X_n$ and $\tilde{A}_1, \tilde{A}_2, ..., \tilde{A}_n$ be defined in the universe sets $X_1, X_2, ..., X_n$, respectively. Let f be a (crisp) mapping from X to a universe Y, $y = f(x_1, x_2, ..., x_n)$. The mapping f for these particular input sets can now be defined as $\tilde{B} = \left\{(y, \mu_{\tilde{B}}(y)) \middle| y = f(x_1, x_2, ..., x_n), (x_1, x_2, ..., x_n) \in X\right\}$, in which the membership function of the image $\tilde{B}$ can be defined (Zimmermann 1991; Tsakiris and Spiliotis 2014) by:

$$\mu_{\tilde{B}}(y) = \sup_{(x_1, x_2, ..., x_n) \in f^{-1}(y)} t\left(\mu_{A_n}(x_1), ..., \mu_{A_n}(x_n)\right) \tag{4}$$

where f^1 is the inverse image of f and t a fuzzy intersection. Mainly the min intersection which is the intersection of the classical logic is widely used.

The implementation of the extension principle gives the opportunity to use a crisp function in which the input variables are fuzzy numbers (Tsakiris and Spiliotis 2016; Kechagias and Papadopoulos 2007).

In most cases, it is preferable to use α-cuts in the fuzzy analysis. If f is a continuous function in the extension principle, the use of α-cuts can be also extended by determining the α-cuts of the function f (Buckley and Eslami 2002; Buckley et al. 2002). Then, based on the min intersection it holds:

$$\begin{cases} f^L\left(\tilde{A}_1,\tilde{A}_2\right)_\alpha = \min\left\{f\left(x_1,x_2\right)\left|x_1 \in \tilde{A}_{1\alpha}, x_2 \in \tilde{A}_{2a}\right.\right\}, \\ f^R\left(\tilde{A}_1,\tilde{A}_2\right)_\alpha = \max\left\{f\left(x_1,x_2\right)\left|x_1 \in \tilde{A}_{1\alpha}, x_2 \in \tilde{A}_{2a}\right.\right\}. \end{cases} \tag{5}$$

In this article concepts from the probability theory, instead of the min-intersection are used and hence, the decision space is reduced (any fuzzy intersection is smaller or equal than the min intersection). In the next section, the relation between copulas and fuzzy intersections will be presented and explained briefly.

As mentioned earlier, instead of the min intersection, several other intersections can be used in the extension principle.

In general, a triangular norm (t–norm for short) (bivariate aggregation function) is a binary operation on the unit interval: $\mathrm{T}\ ([0,1]\times[0,1]\to[0,1])$ which is associative, symmetric, it satisfies the monotonicity and it has neutral element 1.

The above set of axioms constitutes the axiomatic skeleton of fuzzy intersections, T-norms. All functions which satisfy the above axioms are fuzzy intersections (e.g. Klir and Yuan 1995). Among them, one characteristic fuzzy intersection is the min intersection, $\mathrm{T_{min}}$ (Eq. 6), and the intersection of bounded difference, $\mathrm{T_{bd}}$ (Eq. 7):

$$\mathrm{T_{min}}(\mathrm{a},\mathrm{b}) = \min(\mathrm{a},\mathrm{b}) \tag{6}$$

$$\mathrm{T_{bd}}(\mathrm{a},\mathrm{b}) = \max(\mathrm{a}+\mathrm{b}-1,\ 0) \tag{7}$$

It holds that every fuzzy intersection has smaller values than the minimum intersection.

3. COPULAS AND FUZZY INTERSECTIONS

Next, the concept of Copulas and their connection with the fuzzy t-norm will be presented in brief.

Definition: A bivariate copula is a function $C:\left[0,1\right]^2 \to \left[0,1\right]$ which satisfies:

$$C\left(x,0\right)=C\left(0,x\right)=0 \textit{ and } C\left(x,1\right)=C\left(1,x\right)=x \text{ for all } x\in\left[0,1\right] \tag{8.1}$$

$$C\left(x_1,y_1\right)-C\left(x_1,y_2\right)-C\left(x_2,y_1\right)+C\left(x_2,y_2\right)\geq 0 \text{ for all } x_1,x_2,y_1,y_2\in\left[0,1\right]$$
$$\text{such that } x_1\leq x_2 \textit{ and } y_1\leq y_2 \tag{8.2}$$

In statistical analysis, a joint distribution H of a pair of random variables (X, Y) with marginals *F* and *G* respectively, can be expressed by $H(x,y)=C(F(x),G(y))$ for each $(x,y)\in[-\infty,\infty]^2$, in which *C* is a copula uniquely determined.

Copulas can be used also in case that the input x, y have fuzzy membership functions (that is, *x, y* take values between zero and one) (Beliakov et al. 2014).

From the definition, in case that the inputs are fuzzy sets it holds, that Copulas not necessarily are symmetric or associative. Copulas are monotone, non-decreasing (and thus are aggregation functions) and verify

$$\mathrm{T_{bd}} \leq \mathrm{C} \leq \mathrm{T_{min}} \tag{9}$$

(and thus, are conjunctive functions) (Beliakov et al. 2014).

From the mathematical definition, it is evident that, an associative and commutative copula is a t-norm and a t-norm which satisfies the Eq. 8.2, is a copula (Näther 2010).

An interesting family of copulas is the Archimedean family. Among important properties of Archimedean copulas, the symmetry and the associativity are included. In this article, from the Archimedean copulas, the Gumbel-Hougaard copula is selected (e.g. Tsakiris et al. 2016), which belongs to Aczel-Alsina family of t–norms (Beliakov et al. 2014):

$$C\left(x,0\right)=e^{-\left(\sqrt[\theta]{\left(-\ln x\right)^{\theta}+\left(-\ln y\right)^{\theta}}\right)},x,y\in\left[0,\ 1\right],\theta\geq 1 \tag{10}$$

Then, the extension principle based on α-cuts will have the following form:

$$\begin{cases} f^{L}\left(\tilde{A}_1,\tilde{A}_2\right)_{\alpha}=\min\left\{f\left(x_1,x_2\right)\middle|C\left(\mu_1\left(x_1\right),\mu_2\left(x_2\right)\right)\geq a\right\}, \\ f^{R}\left(\tilde{A}_1,\tilde{A}_2\right)_{\alpha}=\max\left\{f\left(x_1,x_2\right)\middle|C\left(\mu_1\left(x_1\right),\mu_2\left(x_2\right)\right)\geq a\right\}. \end{cases} \tag{11}$$

Instead of the individual α-cuts of each parameter, the bivariate boundary of each α-cut is now the curve of copulas with membership function equal to the examined cut. Let for instance, a function with two independent variables. The 0.1-cut according to which the boundaries of the α-cut are examined, by following the min-intersection, will be the rectangular with membership functions greater or equal to 0.1. In the case of Gumbel-Hougaard copulas with *θ = 2,* the decision space is reduced as can be seen from Fig. 1. It should be noted also, that when the parameter θ increases the copulas leads to the min intersection and hence the Eq. 11 leads to Eq. 5.

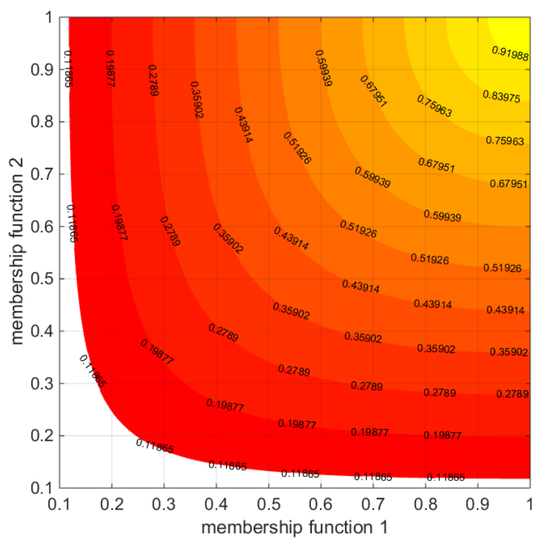

Figure 1. The 0.1-cut for two variables and several bivariate α-cuts based on the Gumbel-Hougaard family of copulas with θ = 2

4. UNCERTAINTY IN FUZZY SETS

The **Q**-equations are used to deal with the uncertainties encountered in the looped water distribution systems. To simplify the calculation procedure, the Hazen -Williams equation is used for the calculation of head losses:

$$h_{f,in} = R_{in} Q_{in}^{\ 1.852},\ R_{in} = \frac{10.7\,L_{in}}{C_{in}^{\ 1.852} D_{in}^{\ 4.87}} \tag{12}$$

in which R_{in} is the hydraulic resistance and C_{in} is the Hazen-Williams roughness coefficient. Also, L_{in}, D_{in} are the length and the internal pipe diameter of the branch *i-n*, respectively.

According to the proposed methodology, initially, we determine the resistance of each pipe as fuzzy number based on the extension principle for a certain number of α-cuts:

$$\begin{cases} R_{in}^{L}\left(x_{1,in}, x_{2,in}\right)_{a} = min\left\{ \dfrac{10.7\,L_{in}}{x_{1,in}^{1.852}\ x_{2,in}^{4.87}} \middle| C\left(\mu_1\left(x_{1,in}\right), \mu_2\left(x_{2,in}\right)\right) \geq \alpha \right\}, \\ R_{in}^{R}\left(x_{1,in}, x_{2,in}\right)_{a} = max\left\{ \dfrac{10.7\,L_{in}}{x_{1,in}^{1.852}\ x_{2,in}^{4.87}} \middle| C\left(\mu_1\left(x_{1,in}\right), \mu_2\left(x_{2,in}\right)\right) \geq \alpha \right\} \end{cases} \tag{13}$$

where $R_{in}^{\ L}$, $R_{in}^{\ R}$ the left and the right hand side of the resistance at pipe in, $x_{1,in}$ an auxiliary variable which indicates the value of the Hazen-Williams coefficient, $x_{2,in}$ the internal diameter, C a selected copula which will be also a t-norm. Also, μ_1, μ_2 are the membership functions of the Hazen-Williams coefficient and the internal diameter, respectively. In this article by taking concepts from the probability theory instead of the min intersection, the Gumbel-Hougaard copulas with θ=2 is used. For illustration purposes, the Π-shaped membership functions of the internal diameters 0.5 and 0.7 m are presented in Fig. 2.

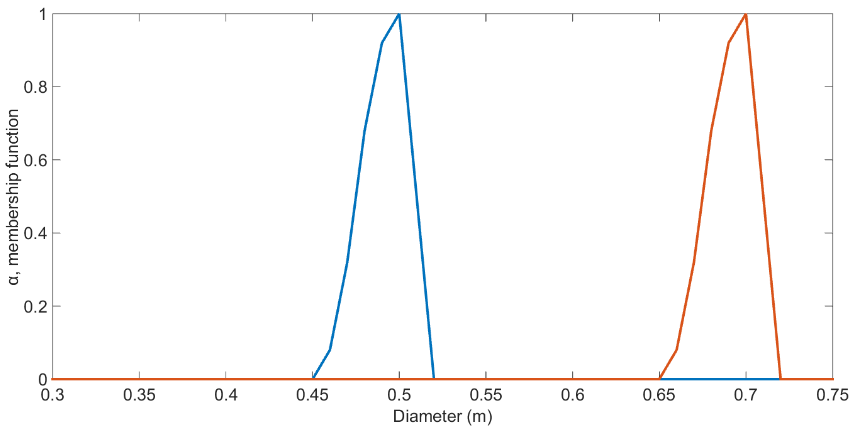

Figure 2. Π-shaped membership functions of the internal diameters of 0.5 and 0.7m.

Alternatively, the resistance can be considered based on initial information or expert knowledge.

Thus, the input variables are fuzzy numbers and the problem is to find the output of the system which will consists also of fuzzy quantities. The system itself is described by a set of precise (crisp) nonlinear equations. In general, this is an ill -constructed problem and several problem depending o each problem itself can be established. Revelli and Ridolfi (2002) proposed the use of an optimisation procedure to determine the minimum and the maximum water flow individually at each branch for each α-cut. Further, Tsakiris and Spiliotis (2017) proposed to produce a simultaneous estimation of all hydraulic variables and their fuzziness. Therefore, an objective function which comprises the weighted sum of the absolute values of the head losses has been proposed. The selection of the weights is based on the principle that each weight is directly proportional to the corresponding flow.

In contrast with the above methodologies, the resistance and its fuzziness for each pipe is determined separately based on a new interpretation of the extension principle with copulas. Thus, during the analysis, the unique set of parameters with fuzziness is the set of the pipe resistances. Then, an individual solution for each pipe in order to determine the range of the flow for each pipe can be implemented. Alternatively, a global membership function as Spiliotis and Tsakiris (2017) proposed can be established:

$$\begin{cases} \min \sum_{in=1}^{L+M-1} \left(\frac{\left| x_{1,in}{}' \right|}{\left(\sum_{in=1}^{L+M-1} \left| x_{1,in}{}' \right| \right)} \right) \cdot x_{2,in}{}' \cdot \left| x_{1,in}{}' \right|^{0.852} \cdot x_{1,in}{}' \left(or \max \right) \\ \sum_{in}^{in \in I(l)} \pm x_{2,in}{}' \cdot \left| x_{1,in}{}' \right|^{0.852} \cdot x_{1,in}{}' = 0, \; l = 1,\ldots,L \\ x_{2,in}{}' \in \widetilde{R_{in\alpha}}, x_{1,in}{}' \left(crisp \right) \\ \sum_{in}^{in \in I_1(m)} \pm x_{1,in}{}' = q_m, \; m = 1,\ldots,M \end{cases} \tag{14}$$

where $\left(\frac{\left| x_{1,in}{}' \right|}{\left(\sum_{in=1}^{L+M-1} \left| x_{1,in}{}' \right| \right)} \right) = w_{in}$ is the weight. It holds: $\sum_{in=1}^{L+M-1} \left(\frac{\left| x_{1,in}{}' \right|}{\left(\sum_{in=1}^{L+M-1} \left| x_{1,in}{}' \right| \right)} \right) = 1$. (14.a)

In Eq. 14, $x_{2,in}'$ is an auxiliary variable which takes values within the corresponding α-cut of the resistance $\tilde{R}_{ina}$ and $x_{1,in}'$ is the flow at pipe *in* (crisp number). The only constraint which comes from fuzzy theory is that the resistance of each pipe must belong to its α-cut. In addition, L is the number of loops and M is the number of nodes. $I_1(m)$ is the set of the pipes that converge to node m and $I(l)$ is the set of the pipes which comprise the loop l.

As mentioned previously, the use of the α-cuts instead of the membership function for corresponding fuzzy sets is adopted. The analysis is based on the crisp **Q**-equations, whereas as fuzzy inputs, the α-cut sets for the pipe resistances are considered. A minimization problem and a maximization problem is solved to determine the left and the right-hand boundaries of the examined α-cut. The process is repeated for a significant number of α-cuts and thus, the membership function of the water flow in the pipes can be reached.

5. NUMERICAL APPLICATION

Let the system of Fig. 3 which consists of two loops, five branches, and four nodes. The system is the one presented by Revelli and Ridolfi (2002), slightly modified. Larger diameters have been selected to fulfil the constraints of max allowable velocity. The Hazen-Williams equation is used for the calculation of head losses. Furthermore, the uncertainty in this example refers not only to the pipe roughness coefficients (as in the original paper), but also to the internal pipe diameters.

Initially, the fuzzy number which corresponds to the pipe resistance is calculated for a discrete number of α-cuts with respect to Eq. 13 (Fig. 4). Comparing the pipe resistances based on the proposed copula approach (Eq. 13) and the widely-used method of α-cuts and the min intersection (Eq. 5), it can be seen, that the proposed methodology leads to smaller uncertainty for the α-cuts belonging to the open interval zero-one (Fig. 4).

The proposed global objective function of the weighted sum of the absolute values of the head losses is examined, by considering crisp numbers for the water consumption at the nodes. As mentioned earlier, the selection of each weight is proportional to the magnitude of flow at each branch. The only inputs with fuzziness are the pipe resistances. The results based on Eq. 14 are shown in Figure 5. The membership functions of the water flow at pipes take a significant non-symmetric shape. However, even if a significant fuzziness is considered at the initial variables, the final fuzziness is substantially reduced.

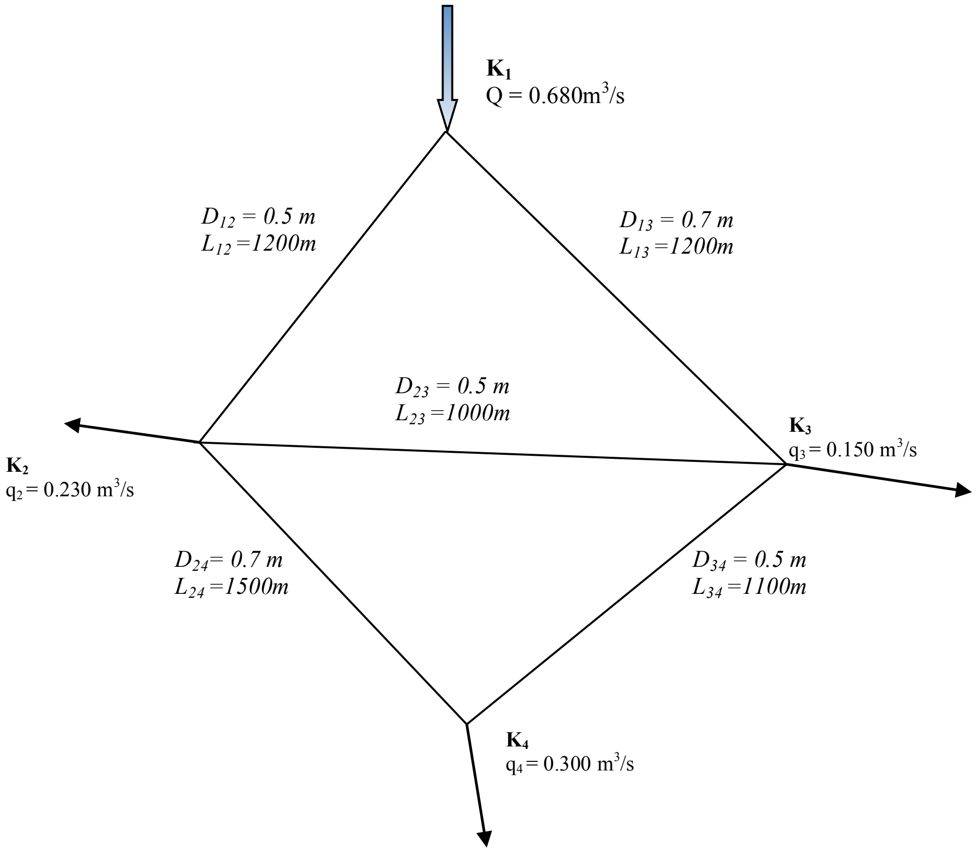

Figure 3. The looped distribution system of the example

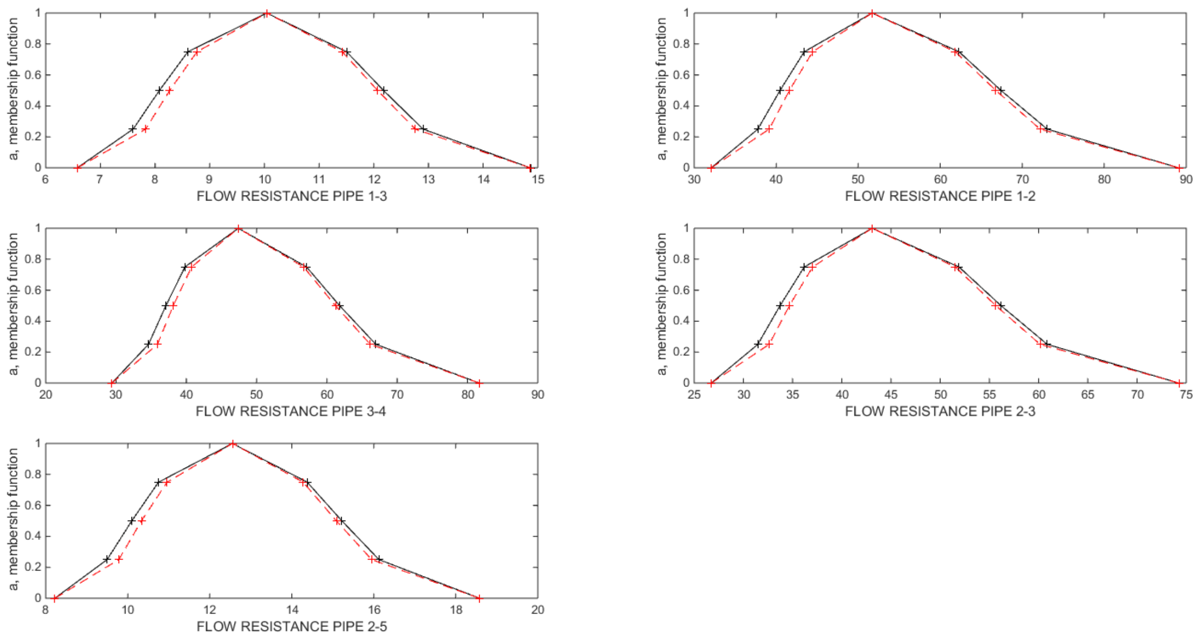

Figure 4. Pipe resistance to flow based on the min intersection (solid line) and the Gumbel-Hougaard copula (dashed line)

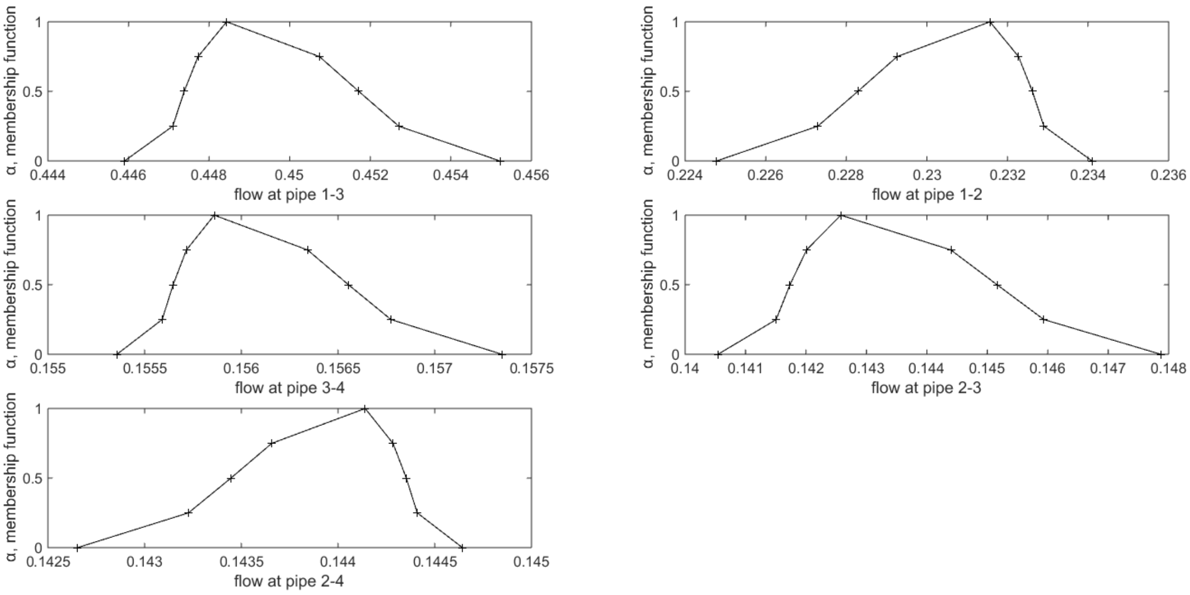

Figure 5. Membership function of flow in each pipe based on the proposed copula approach

6. CONCLUDING REMARKS

It is concluded that the proposed copula approach used in the design of looped water distribution systems, simplifies the uncertainty analysis and facilitates the incorporation of information related to the resistance to flow. Further, it is observed that the copula approach leads to the reduction of fuzziness when compared to the min intersection approach. The study presented in this paper shows also that the copula approach can be also used in the fuzzy analysis although its origin comes from the probability theory.

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