European Water 58: 513-520, 2017.
© 2017 *E.W. Publications*

Relationship between Hazen-William coefficient and Colebrook-White friction factor: Application in water network analysis

M. Niazkar[1], N. Talebbeydokhti[2] and S.H. Afzali[1*]
[1] *School of Engineering, Department of Civil and Environmental Engineering, Shiraz University, Shiraz, Iran*
[2] *School of Engineering, Department of Civil and Environmental Engineering, Head of Environmental Research and Sustainable Development Center, Shiraz University, Shiraz, Iran*
[*] *e-mail: afzali@shirazu.ac.ir*

Abstract: Although Darcy-Weisbach (D-W) equation has been accepted as a standard resistance equation in pressurized flow, some researchers and engineers still prefer to utilize Hazen-Williams (H-W) equation for analyzing water distribution networks (WDNs) in practice. The main difference between roughness coefficients of these two resistance equations is that D-W friction factor varies with Reynolds number of flow field while H-W coefficient is usually considered as a fixed value for a specific material. In this paper, discrepancies between solving three pipe networks using these two resistance equations are investigated. These networks were solved using two hydraulic solver named gradient algorithm and finite element method assuming different values for roughness values. In order to compare the results, equivalent friction factors based on these two resistance equations were required to be used. In this regard, several methods for computing equivalent roughness values were selected from the literature. The first category uses Reynolds number and pipe diameter data for converting D-W friction factor to H-W coefficients and vice versa. On the other hand, the second one is exclusively a function of pipe diameter. The obtained results demonstrate that errors of solving networks with H-W coefficients are not significant in comparison with the results obtained by the D-W equation. More importantly, the method, which only exploits pipe diameter for roughness conversion, is appeared to be not reliable for some cases considered in this paper.

Key words: Resistance equation, Darcy-Weisbach friction factor, Hazen-William coefficient, water distribution system, turbulent flows

1. INTRODUCTION

Hazen-William (H-W) and Darcy-Weisbach (D-W) formulas are two of the most common resistance equations in pressurized flow. Although the application of the former one is quite widespread in practice, especially in the United States, the latter one has much more reasonable background and acceptability in academic literature (Travis and Mays, 2007; Elhay and Simpson, 2011). The empirical H-W equation utilizes a crisp coefficient (C_{HW}) for each type of pipe material. However, the friction factor (f) of the dimensionally-consistent D-W equation is a function of material properties (absolute roughness, ε) and Reynolds number (Re) in turbulent flows, the most common flow regime in water networks. In spite of numerous applications of these resistance equations in water distribution network (WDN) problems, only a few studies investigated the accuracy of such applications (Lion, 1998; Christensen et al., 2000; Bombardelli and Garcia, 2003; Uribe et al., 2015). In this regard, much more studies are required to be conducted focusing on the application of these crucial equations and also relationships between C-W and D-W equations in pipe network analysis.

In this paper, the discrepancies between solving three sample pipe networks using these two resistance equations are investigated. Moreover, the performance of different available relationships between C_{HW} and f in analyzing water networks are studied. These analyses are conducted in the whole range where the C-W equation is valid. Furthermore, two different hydraulic solvers are utilized to determine the performance of different relations for WDN analysis. The results overall indicate that application of H-W equation and the relationships converting C_{HW} to f, which considers both pipe diameter and Reynolds for conversion, yield to closer results to when D-W friction factor is used. Moreover, H-W equation with relationship converting to D-W friction factor, which exclusively takes into account pipe diameter for the conversion, should be used with more caution.

2. RELATIONSHIPS BETWEEN H-W AND D-W COEFFICIENTS

The H-W and D-W resistance equations relate head loss (h_L) of a typical flow through an arbitrary pipe as shown in Eq. 1:

$$h_L = \frac{10.67L}{C_{HW}^{1.852} D^{4.87}} Q^{1.852} = \frac{8fL}{\pi^2 g D^5} Q^2 \quad (1)$$

where L is pipe length, π is pi number, g is gravitational acceleration, D is pipe diameter, and Q denotes pipe flow rate. In Eq. 1, the first and second right hand-side terms are H-W and D-W equations, respectively. The multiplier 10.67 in H-W equation is only valid when D and L are substituted in SI values (Larock et al., 2000).

As implied in Eq. 1, different resistance equations are supposed to yield into a unique head loss for a specific flow. Since different resistance equations have different multipliers and exponents for involving parameters, the aforementioned closure requires use of equivalent flow resistance coefficients.

In essence, relationships in which convert C_{HW} to equivalent f may be classified into three major groups. The first category includes relations that require pipe diameter and flow Reynolds number for this conversion; whereas the second category exclusively utilizes pipe diameter information. The equations of the first category include Liou's (1998) and Locher's (2000) equations while Travis and Mays' (2007) equation can be classified in the second category. These equations are shown in Eq. 2 to Eq. 4, respectively.

$$f = 133.80 C_{HW}^{-1.85} \mathrm{Re}^{-0.148} D^{-0.0158} \upsilon^{-0.148} \quad (2)$$

$$f = 1016.610 C_{HW}^{-1.85} \mathrm{Re}^{-0.148} D^{-0.0158} \quad (3)$$

$$\varepsilon = D(3.320 - 0.021 C_{HW} D^{0.01})^{2.173} \exp(-0.04125 C_{HW} D^{0.01}) \quad (4)$$

The third category may be relations which directly relate C_{HW} to ε and vice versa without requiring any pipe-diameter information. For instance, Eq. 5 and Eq. 6 are the examples of such equations which are obtained using Excel-embedded regression tools (Niazkar and Afzali, 2015, 2016) based on available and reliable data including C_{HW} and their equivalent ε (Liou, 1998; Valiantzas, 2008; Uribe et al., 2015) listed in Table 1. As shown, the range of applicability of these equations are $100<C_{HW}<150$ and $0.0015\text{mm}<\varepsilon<1.5200\text{mm}$.

$$C_{HW} = -348.15\varepsilon^6 + 1436.1\varepsilon^5 - 2132\varepsilon^4 + 1315.9\varepsilon^3 - 224\varepsilon^2 - 85.538\varepsilon + 149.32 \qquad R^2 = 0.94 \quad (5)$$

$$\varepsilon = 2\times10^{-7} C_{HW}^5 - 0.0002 C_{HW}^4 + 0.0396 C_{HW}^3 - 5.0397 C_{HW}^2 - 5.0397 C_{HW}^2 + 319.04 C_{HW} - 825.6 \qquad R^2 = 0.95 \quad (6)$$

3. APPLICATION AND RESULTS

Calculating flow roughness is one of the inevitable steps in WDN analysis. Since this analysis requires iterations where flow rate and Reynolds number may change in each iteration, those flow resistance coefficient, which varies with flow regime variation, should be revised in each iteration of such analyses. In this research, the impact of resistance equation selection on pipe network analysis is the main focus. The most common resistance equations, i.e., H-W and D-W equations, are utilized for this purpose. Two hydraulic solvers, invariantly called gradient algorithm and finite element method (FEM), are used to solve three sample pipe networks selected from the literature. For each network, five scenarios described later in this paper are considered and in each one, the network was solved assuming twenty two C_{HW} with their equivalent ε selected from the literature (Liou, 1998; Valiantzas, 2008; Uribe et al., 2015). In each WDN analysis, similar pipe material was assumed for all pipes. In other words, it was assumed that all pipes have the same C_{HW} while different values of C_{HW}, ranging from 100 to 150 (Table 1), were utilized in different WDN analyses. This range includes a variety of different pipe materials in practice since proper application of H-W equation is for $100<C_{HW}<160$ (Diskin, 1960).

Table 1. Equivalent C_{HW} and ε used in D-W friction factor calculation

Item	Pipe materials	C_{HW}	ε(mm)	Reference
1	Uncoated, new cast iron	120	0.2781	Liou (1998)
2	Uncoated, new cast iron	129	0.1643	Liou (1998)
3	Uncoated, new cast iron	121	0.3007	Liou (1998)
4	Coated, very straight, no specials	144	0.0579	Liou (1998)
5	Coated, Bonn service main, new	114	0.8534	Liou (1998)
6	Coated, Bonn service main, new	111	1.0363	Liou (1998)
7	Coated, well laid, new cast iron	146	0.0488	Liou (1998)
8	Coated, well laid, new cast iron	145	0.0570	Liou (1998)
9	Coated, Danzing main, new cast iron	131	0.2846	Liou (1998)
10	Uncoated, new cast iron	115	0.9498	Liou (1998)
11	Coated, straight, no special, new	140	0.1598	Liou (1998)
12	Coated, Rochester main, new cast	129	0.3109	Liou (1998)
13	Coated, Rosemary siphon, new cast	142	0.0975	Liou (1998)
14	Coated, Edinburgh main, new cast	112.3	1.3411	Liou (1998)
15	Coated sheet iron, riveted	133	0.0908	Liou (1998)
16	Tuberculated Rosemary siphon, cast	112	1.4630	Liou (1998)
17	Cleaned Rosemary siphon, cast iron	142	0.1097	Liou (1998)
18	Exposy coasted steel	145	0.0280	Valiantzas (2008)
19	Plain steel, new	130	0.2030	Valiantzas (2008)
20	Concrete	100	1.5200	Valiantzas (2008)
21	PVC	150	0.0015	Uribe et al. (2015)
22	Cast iron	140	0.1000	Uribe et al. (2015)

In order to assess the performance of H-W equation and the available relationships between C_{HW} and f, each pipe network was solved for five different scenarios under steady-state condition. These scenarios are presented as following:

1. In the first scenario, the implicit Colebrook-White (C-W) formula is used in the process of analyzing sample network. Since this formula is generally accepted as a standard equation for computing D-W friction factor in the literature, the results of this scenario is considered as the benchmark solution.
2. The H-W equation is considered as flow resistance equation in the second scenario. In order to provide a rational comparison, equivalent C_{HW} listed in Table 1 are used for each pipe material in this scenario.
3. Liou's (1998) equation is used to convert C_{HW} to f. Since this equation requires pipe diameter and Reynolds number for conversion, it is used in each iteration of hydraulic solvers. Finally, it is emphasized that the D-W equation is used as the resistance equation in this scenario.
4. The fourth scenario is similar to the third one except that in this scenario, Locher's (2000) equation is used instead of Liou's (1998) equation.
5. Unlike the third and fourth scenarios, the second type of relationships between C_{HW} and f, i.e.,

Travis and Mays's (2007) equation, is utilized in this scenario. Since the second category relates ε with C_{HW} by using only pipe diameter information, there is no need to use this equation in each iteration of WDN analysis whereas f is evaluated based on absolute roughness and Reynolds number in each iteration.

It should be noted that unknown hydraulic heads are calculated considering H-W equation in the second scenario in both gradient algorithm and FEM while D-W equation is used in other scenarios. Furthermore, C_{HW} is converted to equivalent f in the main calculation in the three last scenarios. As hydraulic head values are computed in the hydraulic solvers in these scenarios, flow rate are computed using H-W and D-W equations in FEM and gradient algorithm, respectively. These h-based methods are coded in MATLAB and Excel spreadsheet and the step-by-step details of these methods and their codes can be found in Niazkar and Afzali's (2017a) paper for interested readers. Finally, the root mean square error (RMSE) and mean absolute relative error (MARE) are used to compare the hydraulic heads obtained in different scenarios with that of the first scenario. These evaluation criteria are shown in Eq. 7 and Eq. 8, respectively.

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}(h_{i,1} - h_{i,j})^2}{N}} \qquad \forall j = 2,3,4,5 \tag{7}$$

$$MARE = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{h_{i,1} - h_{i,j}}{h_{i,1}}\right| \qquad \forall j = 2,3,4,5 \tag{8}$$

where $h_{i,1}$ and $h_{i,j}$ are hydraulic heads at ith node computed in the first and jth scenario, respectively, and N is total number of nodal points in pipe network.

The first pipe network is a small one having seven pipes, six nodes, and a reservoir (Niazkar and Afzali, 2017b). The variation of RMSE computed for the second to fifth scenarios using FEM and gradient algorithm are depicted in Figure 1. As shown, RMSE for the second and third scenarios are close for various C_{HW} while RMSE values for the fourth scenario are sometimes smaller and sometimes larger than the ones calculated for the second scenario. On the other hand, the worst RMSE values are obtained for the fifth scenario for almost all C_{HW} values. This indicates that the method in which uses pipe diameter for converting C_{HW} into equivalent f yield to larger RMSE in comparison with other scenarios.

The second WDN includes 74 pipes, 48 nodes, and two reservoirs (Chin et al., 1978). Figure 2 illustrates RMSE values calculated for different scenarios for different C_{HW} values. According to Figure 2, Travis and Mays' (2007) equation used in the fifth scenario achieved the worst results based on the RMSE values. The discrepancy between hydraulic heads of the first and the fifth scenarios in terms of RMSE is more significant for the lowest C_{HW} values and it decreases with the increase of C_{HW} values. Achieving large RMSE values for the fifth scenario indicates that using relationships between C_{HW} and f which exclusively utilizes pipe diameter for conversion may be unreliable in WDN analysis. Moreover, larger RMSE values overall is achieved when FEM is utilized for solving the second WDN, especially in the fifth scenario.

A real pipe network which consists of 91 pipes, 65 computational nodes, and one reservoir (Arsene and Gabrys, 2014) was solved as the third WDN. The accuracy of different scenarios is compared in Figure 3 for the third network. Based on Figure 3, considered scenarios overall yield to larger RMSE values for C_{HW} values lower than 130 while the accuracy of these scenarios increase for C_{HW} values larger than 130. Although the range of RMSE values depicted in Figure 3 is relatively much lower than the one shown in Figure 2, the fifth scenario in both WDNs relatively result larger RMSE values. In Figure 3, the fourth scenario also achieves the highest RMSE for some cases with C_{HW} lower than 130.

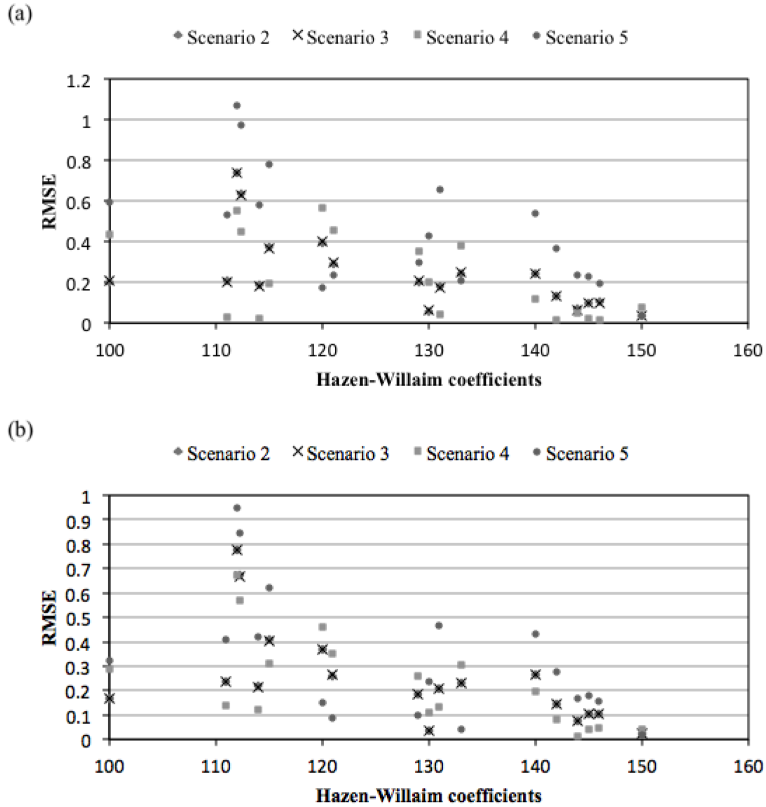


Figure 1. Variation of RMSE for different scenarios computed using (a) FEM and (b) gradient algorithm for the first pipe network.

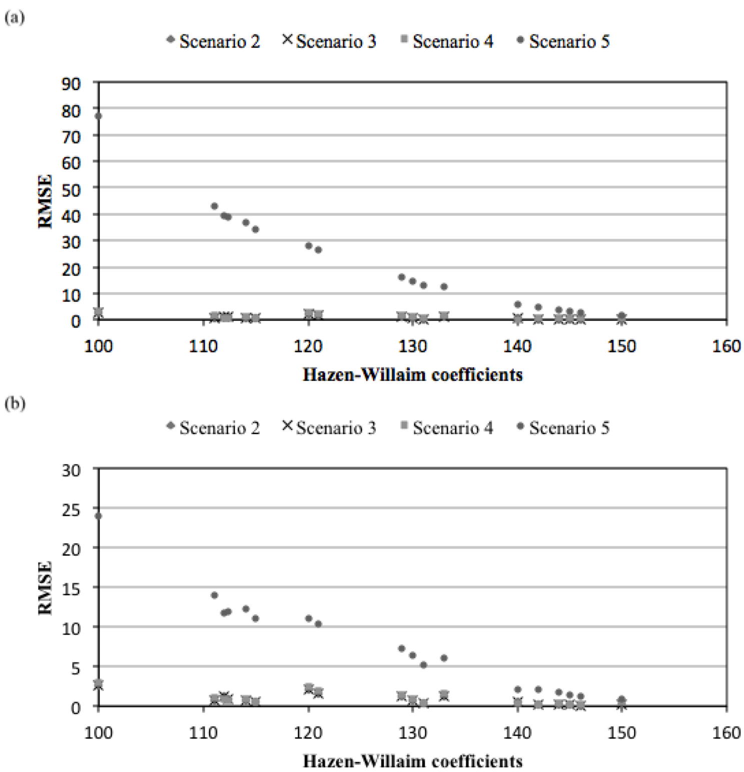


Figure 2. Variation of RMSE for different scenarios computed using (a) FEM and (b) gradient algorithm for the second pipe network.

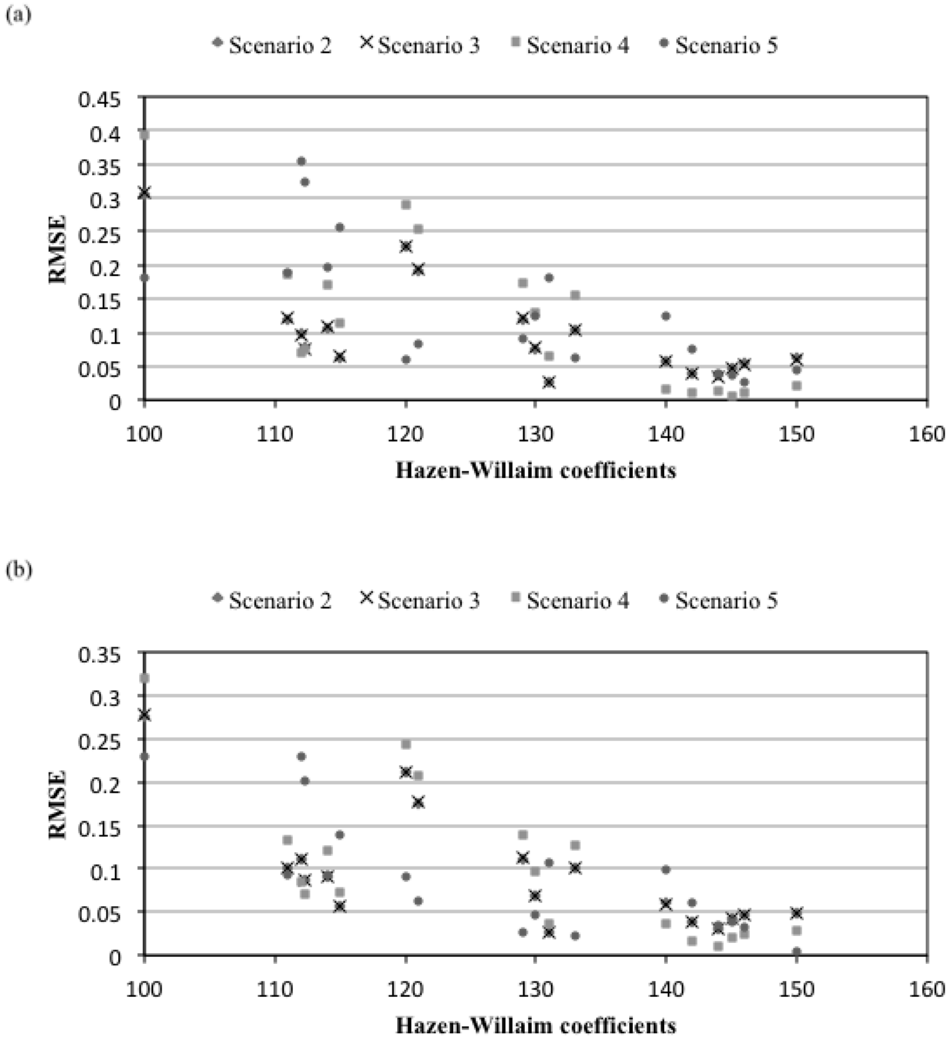
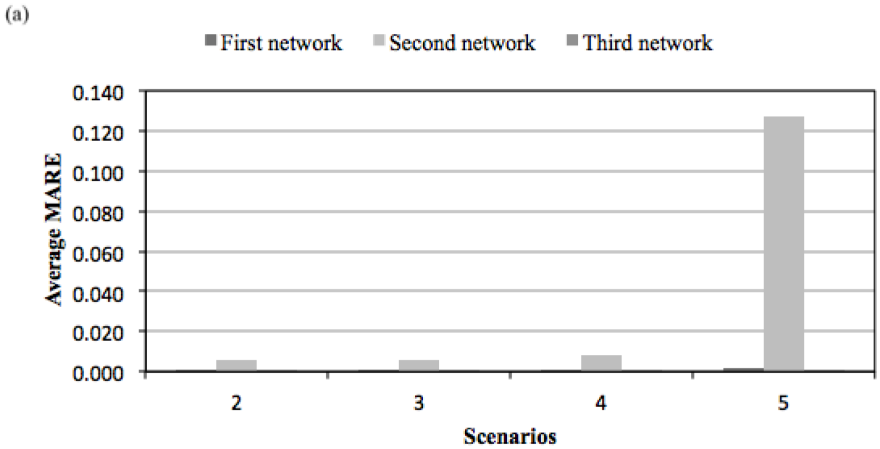


Figure 3. Variation of RMSE for different scenarios computed using (a) FEM and (b) gradient algorithm for the third pipe network.

In order to determine a comprehensive comparison between these scenarios, the average of RMSE and MARE values achieved for different C_{HW} values are computed for each network and each scenario. The average of RMSE values for the three WDNs solved by FEM and gradient algorithm are listed in Table 2. Furthermore, the average of MARE values computed for different scenarios are shown in Figure 4. Table 2 indicates that both h-based methods achieve quite similar average RMSE values for the first and third pipe networks. However, achieved RMSE in the fifth scenario using FEM is approximately three times larger than the one obtained in the same scenario using gradient algorithm. According to Table 2 and Figure 4, solving the three WDNs demonstrates that the second and third scenarios overall achieve the closer results to the first scenario whereas the fifth scenario yield to significant errors, especially for the second network. In other words, solving WDN using H-W equation (second scenario) or using Liou's (1998) equation for converting C_{HW} to f lead to closer results to when D-W equation is utilized. Additionally, application of Travis and Mays's (2007) equation for converting C_{HW} to f in analyzing WDNs causes significant errors for various C_{HW} values, especially in the second network. Since the application of H-W equation and the relationships, which uses both D and Re for converting C_{HW} to f, to WDN analysis yield to results with MARE<0.01 on average, it can be concluded that these equations performs acceptably accurate. On the other hand, solving WDN using relationships, which exclusively uses D for converting C_{HW} to f, are appeared to be unreliable for some cases.

Table 2. Average RMSE values calculated for different scenarios

FEM (average RMSE)	Scenarios			
	2	3	4	5
First network	0.245	0.243	0.220	0.451
Second network	0.871	0.870	1.193	22.352
Third network	0.101	0.101	0.120	0.136
Average	0.405	0.405	0.511	7.646
Gradient algorithm (average RMSE)	**Scenarios**			
	2	**3**	**4**	**5**
First network	0.250	0.249	0.230	0.326
Second network	0.861	0.861	0.986	7.806
Third network	0.094	0.094	0.100	0.090
Average	0.402	0.401	0.439	2.741



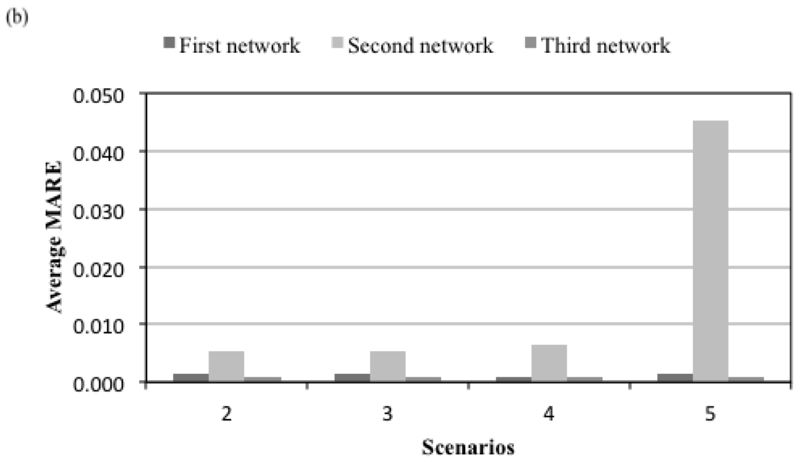


Figure 4. Average MARE values calculated for different scenarios using (a) FEM and (b) gradient algorithm.

4. CONCLUSION

In this research, the applications of H-W equation and relationships which convert C_{HW} to f in analyzing WDN are investigated. Regarding the importance of the subject in WDN literature, three pipe networks were solved considering five scenarios for different values of C_{HW} using two h-based hydraulic solvers. Analyzing pipe networks for 660 different cases in this study shows that H-W equation and the relationships, which uses both pipe diameter and Reynolds number for converting C_{HW} to f, overall yield to acceptably close results to when D-W equation is used. However, application of relationship, which uses pipe diameter for converting C_{HW} to f, do not achieve accurate results for some cases, especially for lower values of C_{HW}.

REFERENCES

Arsene, C. T., Gabrys, B., 2014. Mixed simulation-state estimation of water distribution systems based on a least squares loop flows state estimator. Appl Math Model. 38(2): 599-619.

Bombardelli, F. A., Garcıa, M. H., 2003. Hydraulic design of large-diameter pipes. J Hydraul Eng. 129(11): 839-846.

Chin, K., Gay, R., Chua, S., Chan, C., Ho, S., 1978. Solution of water networks by sparse matrix methods. Int J Numer Meth Eng. 12(8): 1261-1277.

Christensen, B. A., Locher, F. A., Swamee, P. K. 2000. Limitations and proper use of the Hazen-Williams equation. J Hydraul Eng. 126(2): 167-170.

Elhay, S., Simpson, A. R., 2011. Dealing with zero flows in solving the nonlinear equations for water distribution systems. J Hydraul Eng. 137(10): 1216–1224.

Larock, B. E., Jeppson, R. W., Watters, G. Z. 1999. Hydraulics of pipeline systems. CRC press, New York.

Liou, C. P. 1998. Limitations and proper use of the Hazen-Williams equation. J Hydraul Eng. 124(9): 951-954.

Niazkar, M., Afzali, S. H. 2015. Application of Excel spreadsheet in engineering education. Proc. of the First International & Fourth National Conference on Engineering Education, 10-12 November 2015, Shiraz University, Shiraz.

Niazkar, M., Afzali, S. H., 2016. Streamline Performance of Excel in Stepwise Implementation of Numerical Solutions. Comput Appl Eng Educ. 24(4): 555–566.

Niazkar, M., Afzali, S. H., 2017a. Analysis of water distribution networks using MATLAB and Excel spreadsheet: h-based methods. Comput Appl Eng Educ. 25(1): 129-141.

Niazkar, M., Afzali, S. H. 2017b. Analysis of water distribution networks using MATLAB and Excel spreadsheet: Q-based methods. Comput Appl Eng Educ. 25(2): 227-289.

Travis, Q. B., Mays, L. W. 2007. Relationship between Hazen–William and Colebrook–White Roughness Values. J Hydraul Eng. 133(11): 1270-1273.

Uribe, J., Saldarriaga, J., Páez, D. 2015. Effects of the Use of Hazen-Williams Equation on Large WDS Planning Models. In: World Environmental and Water Resources Congress, 881-889.

Valiantzas, J. D. 2008. Explicit power formula for the Darcy–Weisbach pipe flow equation: application in optimal pipeline design. J Irrig Drain E-Asce, 134(4): 454-461.