

Generalized numerical and nomographic solutions of simple pipe flow problems

J. G. Sakkas

School of Engineering, Democritus University of Thrace, P.O.B. 194, 58100 Giannitsa, Greece

e-mail: sakkas.ioa@gmail.com

Abstract: The relation formulated by Swamee and Jain through the combination of the Darcy-Weisbach, on the one hand, and the Colebrook-White equation, on the other, in order to eliminate the Darcy-Weisbach resistance coefficient f , was placed in dimensionless form in two different ways. Each of the new equations contains three dimensionless variables and thus it can be depicted on a single diagram as a family of one-parameter curves. Each of the two equations yields a direct solution to simple pipe flow problems of category 2 and an iterative solution to problems of the other two categories while the corresponding diagrams yield each a direct solution to flow problems of categories 1 and 2 the first and of categories 2 and 3 the second. Regarding the iterative solution of the new equations, an appropriate technique was devised for the direct computation of the correction required in order to proceed from one to the next iteration. Convergence of the iterative process is rapid and the solution is as accurate as the original Colebrook-White equation warrants.

Key words: Colebrook-White equation, Darcy-Weisbach equation, nomographic solution, numerical solution, steady pipe flow

NOTATION

A = cross sectional area of pipe, [m²]
 D = inner diameter of pipe, [m]
 f = the Darcy - Weisbach friction factor, [-]
 g = acceleration due to gravity, [= 9.81 m/s²]
 h_f = linear head losses, [m]
 k = height of roughness elements in pipe wall, [m]
 L = length of pipe, [m]
 Q = discharge of pipe, [m³/s]
 Q_D = dimensionless discharge, [-]
 Q_k = dimensionless discharge, [-]
 Q_T = square root of dimensionless discharge Q_k, [-]
 R = Reynolds number, [-]
 R_D = conceptual or modified Reynolds number, [-]
 R_k = conceptual or modified Reynolds number, [-]
 S_f = slope of energy grade line, [m/m] or [-]
 V = average flow velocity in the pipe, [m/s]
 Δ = a symbol of difference or change in the value of a variable,
 δ = square root of relative roughness of pipe ε, [-]
 ε = relative roughness of pipe, (ε = k/D), [-]
 ν = kinematic viscosity of flowing fluid, [m²/s]

1. INTRODUCTION

1.1 Definitions and basic equations

The term simple pipe refers to a flow conduit where the existing energy losses are due only to wall friction, i.e., there exist only linear energy losses. This occurs either because in a significant

length of the conduit no local or minor losses are present or the latter, if they appear, constitute, in relation to the linear energy losses, an essentially small percentage of the total head losses. In this case, and in order to facilitate the computation, the minor head losses are neglected.

A better approximation to this problem is obtained by an increase of the linear head losses through a reasonable increase of the actual length of the conduit by the so-called equivalent length, that is, that imaginary length of the same conduit in which the amount of linear head losses produced is equal to the sum of all minor head losses existing in the actual length of the conduit under consideration (see e.g. Streeter, 1979: p. 245; King and Brater, 1963: p. 6.18).

For the solution of flow problems through a closed conduit the Darcy-Weisbach equation is mainly used:

$$h_f = f \frac{L V^2}{D 2g} \quad (1)$$

where h_f = the linear head losses, [m], in the (total) length of the conduit (= actual + equivalent), f = the Darcy-Weisbach friction factor, [-], L = the (total) length of the conduit, [m], D = the (internal) diameter of the conduit, [m], V = the mean flow velocity in the conduit, [m/s], that is:

$$V = \frac{Q}{A} \quad (2)$$

where Q = the flow discharge through the conduit, [m³/s], A = the cross-sectional area of the circular pipe, [m²]:

$$A = \frac{\pi D^2}{4} \quad (3)$$

and g = the acceleration due to gravity [= 9.81 m/s²].

The Darcy-Weisbach friction factor, f , may be determined through the Colebrook-White equation (see e.g. Chadwick and Morfett, 1993: p. 99, Streeter, 1979: p. 234, Terzidis, 1982: p. 278).

$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{k}{3.72D} + \frac{2.52}{R\sqrt{f}} \right) \quad (4)$$

where k = the height of the roughness elements of the conduit walls, [m], and R = the Reynolds number of the flow determined by the relation:

$$R = \frac{VD}{\nu} \quad (5)$$

where ν = the kinematic viscosity of the fluid, [m²/s].

The five algebraic equations, Eq. 1 through 5, contain in total 11 variables which can be classified into three categories:

- a) L, g, k, ν
- b) V, A, f, R
- c) h_f, Q, D

The values of the variables in category (a) are known as being the data of the problem under consideration. The values of the variables in category (b) are determined through the set of Eqs. 2 to

5 when the values of the variables appearing in the right hand side of these equations are known. Especially, for the variable f it is necessary to use an iterative procedure for the numerical solution of Eq. 4 due to the complexity of its form. Finally, for the determination of the values of the variables in category (c) only one relation is available, Eq. 1. In this case, the values of two out of the three remaining variables must be given in order to obtain one explicit and unique solution to the problem, that is the value of the third variable.

Therefore, three types of flow problems through simple pipes arise as is shown in Table 1.

Table 1. Types of flow problems through simple pipes.

Type of problem	Data given	Data required
1	(L , g , k , v), Q , D	h_f
2	(L , g , k , v), D , h_f	Q
3	(L , g , k , v), h_f , Q	D

Various numerical and graphical methods and procedures have been proposed for the solution of the problems above using the set of Eqs. 1 to 5, as will be stated in brief in the following section.

In this paper, one more numerical method, as well as another graphical procedure, are proposed for an easy and fast solution of all three types of flow problems under consideration.

1.2 Existing methods of solving simple pipe flow problems

1.2.1 Numerical solution of simple pipe flow problems

Pipe flow problems of type 1 are solved directly because the given data permit: the immediate determination of the values of variables A , V and R , then the computation of the friction factor f through an iterative procedure and, finally, the direct determination of the value of h_f .

The other two types of problems are solved in a numerical way only through an iterative procedure. With the assumption of an approximate value of the discharge Q or of the diameter D , respectively, for the second or the third type of problem, both types 2 and 3 of flow problems are converted to problems of type 1. The iterative procedure is continued until the computed value of head loss h_f - approaches sufficiently the given value.

1.2.2 Exact numerical solution for the flow problem of type 2

The Colebrook-White equation for the determination of the value of f , Eq. 4, is of implicit form, thus an iterative procedure is required for its numerical solution. Therefore, various investigators attempted to simplify it through its linearization or to completely substitute it with other equivalent expressions which might contain only known data of the problem. Among these attempts of outstanding importance is the work of Swamee and Jain (1976). Through appropriate transformations of Eqs. 1, 4 and 5 (see e.g. Streeter, 1979: p. 240) the workers above ended in the formulation of the following relation

$$\frac{Q}{D^2 \sqrt{gDS_f}} = -\frac{\pi}{\sqrt{2}} \log \left[\frac{k}{3.72D} + \frac{1.78v}{D\sqrt{gDS_f}} \right] \quad (6)$$

where $S_f = h_f / L$ is the slope of the energy grade line or the energy (or head) loss per unit length. This equation yields a direct solution to flow problems of type 2.

One analogous relation, alternative form of Eq. 6, was formulated a little earlier by Barr (1975).

Also, an analogous to Eq. 6 relation, but with the flow velocity V instead of the discharge Q as the main variable of the problem, was formulated by Ackers in the year 1958 (Ackers, 1958). This relation was used to prepare a set of dimensional Tables and diagrams which facilitate the solution of all three types of pipe flow problems. These computation aids have been published by the Thomas Telford Foundation (Hydraulics Research Ltd., 1983a, 1983b).

1.2.3 An approximate expression for the Darcy-Weisbach friction factor f

In order to facilitate the determination of the value of the Darcy-Weisbach friction factor, f , various approximate relations have been developed out of which only the following one is reproduced here

$$f = \frac{0.25}{\left[\log \left(\frac{k}{3.7D} + \frac{5.74}{R^{0.9}} \right) \right]^2} \quad (10^{-6} \leq k/D \leq 10^{-2}, \quad 5 \times 10^3 \leq R \leq 10^8) \quad (7)$$

This relation was developed by Swamee and Jain (1976) and it is valid for the range of values of the parameters k/D and R given above with a relative error, in comparison with Eq. 4, within the limits $\pm 1\%$.

1.2.4 Nomographic solutions of the Darcy - Weisbach equation

In order to facilitate the solution of flow problems in simple pipes various diagrams or nomograms have been prepared. Out of these the most well known and used is the Moody diagram which is a graphical depiction of the Colebrook-White equation, Eq. 4, and thus it yields the value of f as a function of the Reynolds number, R , and the relative roughness, k/D . Hence, this diagram leads to a direct solution of flow problems of type 1 and, also, it facilitates the iterative solution of flow problems of the other two types.

Various improvements have been made to the Moody diagram by the addition of appropriate scales which permit the direct solution of the other two types of flow problems. Thus, the Moody - Daily - Harleman diagrams (Daily and Harleman, 1966) yield a direct solution to flow problems of types 1 and 2. The same result is also obtained by the generalized Rouse diagram (Rouse, 1943, 1950). These diagrams include four basic scales. It must be stressed that Rouse constructed his generalized diagram independently and one year earlier than Moody (see Rouse, 1943, Moody, 1944).

The most improved and intricate Moody-Terzidis diagram (Terzidis, 1982) contains six logarithmic scales and yields a direct solution to all three types of simple pipe flow problems.

It is obvious that all diagrams with the ability to yield a direct solution to one or two types of flow problems can also be used for solving problems of the remaining types by using the iterative procedure. Yet, the more scales they contain the more the difficulty of using them and the less the accuracy of the solution obtained.

In the year 1950, Powell (1950) obtained the construction of a dimensionless diagram of the Darcy-Weisbach and Colebrook-White equations in which each of the three main variables Q , D and $S_f = h_f / L$ appear autonomously (in the form of a dimensionless mononym). This diagram, [later extended by Asthana (1974)] makes possible the direct solution of all three types of simple pipe flow problems.

Finally, the Hydraulics Research Station in Great Britain, combining the Darcy-Weisbach and the Colebrook-White equations, formulated a relation analogous to Eq. 6, but expressed as a function of the flow velocity V . On the basis of this relation, they constructed dimensional diagrams and tables which, for only one value of the kinematic viscosity ν and for certain values of the

roughness height k , yield directly the value of a main variable [out of Q (or V), D and S_f] when the values of the other two variables are given (Hydraulics Research Ltd., 1983a, 1983b).

1.2.5 Difficulties in designing a nomographic solution

The relation in Eq. 6 contains in total six variables out of which the values of the three ones g , k and v are in general known while the other three Q , D and S_f constitute the main hydraulic variables. Out of these only one might be the solution of the problem under consideration. On the other hand, since the variable S_f is dimensionless and in Eq. 6 it always appears in conjunction with the known parameter g , it is considered useful to introduce in their place another variable $S = g S_f$, thus reducing the number of variables to five. Now, since all variables in Eq. 6 are expressed dimensionally with only two dimensions, L and T , the number of dimensionless variables that can be formed is, according to the π -theorem, three. If, on the other hand, one pursues to form a dimensionless equation giving a direct solution to each of the three types of simple pipe flow problems (or, in other words, to each of the three main hydraulic variables), then each one of them must appear as the unique main variable in a dimensionless variable. This means a priori that none of them can appear as a characteristic variable of non-dimensionalization.

Therefore, it remains as the only possibility the employment of k and v as the characteristic variables of non-dimensionalization. However, their numeric values are in general very small and this entails some difficulties in designing the nomographic solution of Eq. 6.

Indeed, application of dimensional analysis to Eq. 6 under the conditions above yields the three dimensionless variables $Q_p = Q / k v$, $\varepsilon = k / D$ and $R_p = k^3 g S_f / v^2$ (i.e. the three dimensionless variables of Powell-Asthana solution). Introducing into these variables the extreme values that the dimensional variables can, in general and by far, assume in practice, the usual range of values of the three dimensionless variables results as shown in Table 2.

Table 2. Extreme values of dimensional variables and of dimensionless ones of Powell - Asthana solution.

Dimensional variable				Dimensionless variable		
Kind	Unit	Value		Kind	Value	
		Minimum	Maximum		Minimum	Maximum
Q	[m ³ /s]	0.001	100	Q _p	7.2*10 ⁴	2.2*10 ¹⁴
D	[m]	0.010	5.0	ε	3.0*10 ⁻⁷	5.0*10 ⁻²
S _f	[-]	0.0001	0.10	R _p	1.4*10 ⁻⁹	7.7*10 ⁵
k	[m]	1.5*10 ⁻⁶	9.1*10 ⁻³	--	--	--
v	[m ² /s]	3.1*10 ⁻⁷	1.5*10 ⁻⁶	--	--	--
g	[m/s ²]	9.81	--	--	--	--

As it appears from Table 2, for the nomographic depiction of the dimensionless form of Eq. 6, 15 cycles of the logarithmic scale on the R_p axis and 10 such cycles on the Q_p axis are required (in the Asthana diagram there exist 23 and 8 cycles, respectively). This, of course, depends on the range of values of the dimensional variables used. Besides, the range of values of the variable Q_p , as it results from the computation, depends on the equation for its determination, that is the dimensionless form of Eq. 6.

Under the conditions above, the clearness of the diagram and, consequently, its accuracy is limited. The deployment into more sheets of paper in order to increase clearness requires a significant number of sheets due to its two-dimensional form. In the writer's opinion, this is the main reason that the Powell-Asthana nomographic solution did not gain an acceptance analogous to its real value.

A significant improvement of this condition can be obtained with the introduction of the dimensionless variable $R_k \equiv \sqrt{R_p} = k^{3/2} \sqrt{g S_f} / v$ by which the length of the corresponding axis is reduced to 8 cycles (instead of 15). Further improvement of the Powell-Asthana diagram is possible

with the introduction of new dimensionless variables. Development of this subject, which is under consideration, will be the topic of another work.

The search for more clear and direct nomographic solutions to the problem under consideration leads to the replacement of the roughness height k with the diameter D as the characteristic length for non-dimensionalization. Then, the following dimensionless variables appear: $Q_D = Q / D v$, $\varepsilon = k / D$ and $R_D = D^{3/2} \sqrt{g S_f} / \nu$, with range of values:

$Q_D = 1.3 \cdot 10^2 - 3.2 \cdot 10^{10}$, $\varepsilon = 3 \cdot 10^{-7} - 5 \cdot 10^{-2}$ and $R_D = 2 \cdot 10^1 - 3.6 \cdot 10^7$, that is axes length for Q_D and R_D , respectively, 8 and 6 logarithmic cycles. Improvement in clearness is evident and significant, however, the possibility of a direct solution of flow problems of type 3 is lost.

Successful treatment of this problem is obtained by the use of another system of dimensionless variables in which the diameter D in dimensionless form constitutes one of the three dimensionless variables of the new dimensionless Swamee-Jain equation obtained.

In this way and for a direct solution of all three types of simple pipe flow problems, two diagrams are required. Both of them show a valuable advantage: Each diagram appears as a long and narrow strip extending diagonally on the sheet of paper. This enables the deployment of the nomogram on two or three at most sheets of paper in order to obtain more than sufficient clearness as well as accuracy.

The development and utilization of the two systems of dimensionless variables, as described above, for the numerical or direct nomographic solution of all three types of simple pipe flow problems constitutes the subject of this paper.

2. The new dimensionless numerical and nomographic solutions

2.1 Derivation of the Swamee-Jain equation

Combining Eqs. 1 to 3 one obtains, as an expression for the discharge Q , the relation

$$Q = \frac{1}{\sqrt{f}} \frac{\pi D^2}{4} \sqrt{2 g D S_f} \quad (8)$$

where $S_f = h_f / L$. Likewise, combining Eqs. 1 and 5 one obtains the relation

$$R \sqrt{f} = \frac{D \sqrt{2 g D S_f}}{\nu} \quad (9)$$

With the introduction of Eq. 9 into the right hand side of Eq. 4 and substitution of the relation thus obtained into Eq. 8 the Swamee-Jain equation is derived:

$$Q = -\frac{\pi}{\sqrt{2}} D^2 \sqrt{g D S_f} \log \left[\frac{k}{3.72 D} + \frac{1.78 \nu}{D \sqrt{g D S_f}} \right] \quad (10)$$

which was presented by Swamee and Jain (1975).

2.2 The first dimensionless Swamee-Jain equation

Applying the dimensional analysis procedure to Eq. 10, using as characteristic (or repeating) variables the diameter D and the kinematic viscosity ν , three dimensionless variables are obtained defined by the relations:

$$Q_D = \frac{Q}{Dv} \quad (a) \quad \varepsilon = \frac{k}{D} \quad (b) \quad R_D = \frac{D\sqrt{gDS_f}}{v} \quad (c) \quad (11)$$

Introducing these relations into Eq. 10, it finally assumes the dimensionless form:

$$Q_D = -\frac{\pi}{\sqrt{2}} R_D \log \left[\frac{\varepsilon}{3.72} + \frac{1.78}{R_D} \right] \quad (12)$$

The dimensionless variable R_D , defined by Eq. 11c, represents a conceptual or modified Reynolds number.

The relation of Eq. 12 contains three dimensionless variables: Q_D , ε and R_D . Therefore, it can be depicted in a unique diagram as a set of one-parameter curves (with the relative roughness ε as a parameter). This diagram, shown in Fig. 1 and characterized by the distinctive name Nomogram G_{12} , yields a direct solution to flow problems of types 1 and 2.

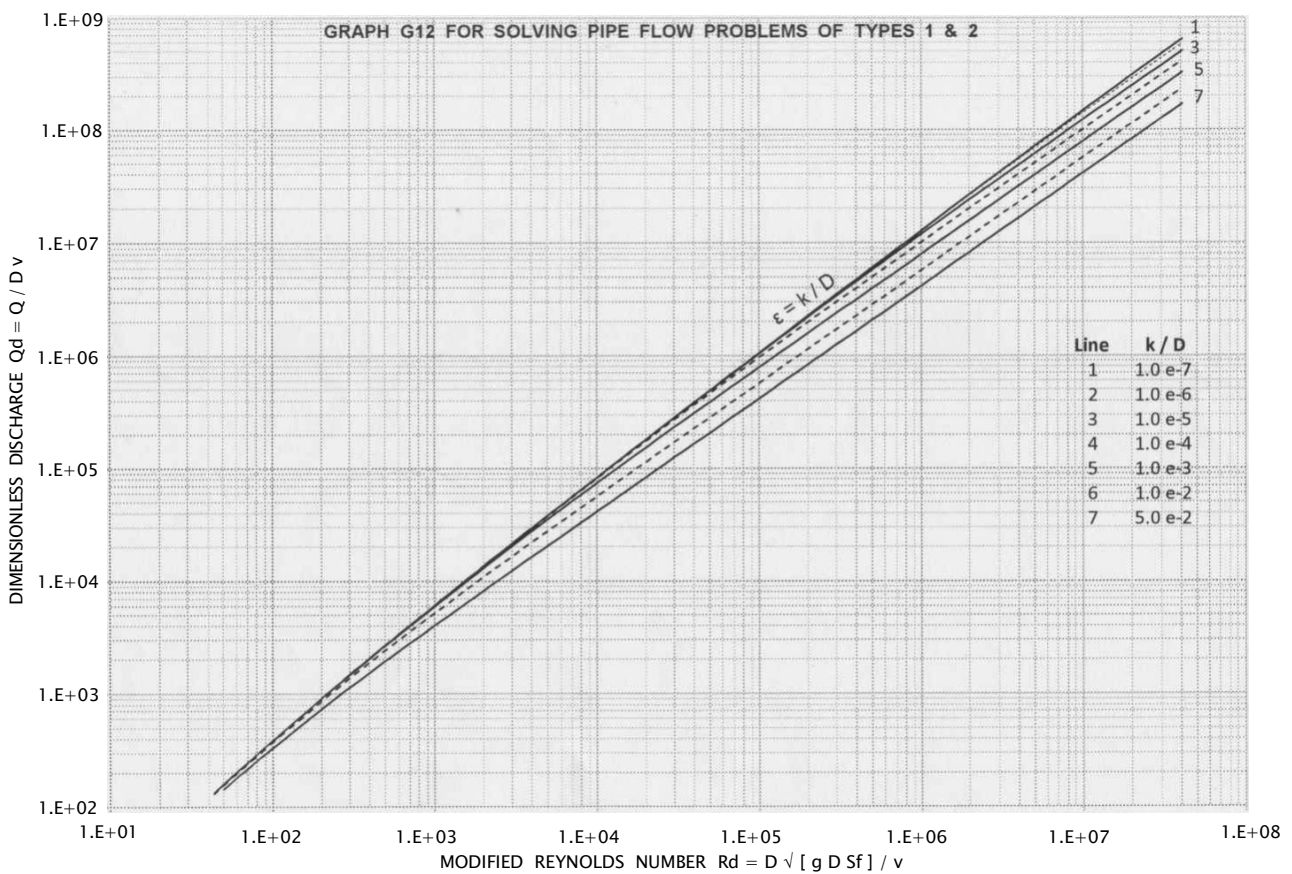


Figure 1. Nomogram G_{12} for the solution of simple pipe flow problems of types 1 and 2.

Combining Eqs. 5, 8 and 11c one concludes that the modified Reynolds number R_D is associated with the real Reynolds number R with the relation:

$$R_D = R \sqrt{\frac{f}{2}} \quad (13)$$

The relation of Eq. 13 was used, in conjunction with the Moody diagram, for defining the range of values of R_D in the construction of the Nomogram G_{12} (Fig. 1).

Furthermore, Eq. 13 explains the simplification occurring in the Moody diagram or its improved derivatives or modifications: the introduction of the modified Reynolds number R_D eliminates the f -

scale incorporating it into the scale of R (here the scale of R_D), thus enabling the introduction of the dimensionless discharge Q_D into the scale of ordinates.

Under the conditions above, Eq. 12 yields a direct analytic solution only to flow problems of type 2. For the numerical determination of the values of the other two main hydraulic variables (the diameter D and the head loss per unit length S_f) the employment of the iterative procedure is required.

On the contrary, the nomographic solution of Eq.12, that is, the diagram G_{12} of Fig. 1 yields a direct solution to flow problems of types 1 and 2 and only for flow problems of type 3 the iterative procedure is required. This last difficulty is overcome by a second diagram to be developed and described in the following paragraph.

Due to the compact form and the linearity of the variables of Eq. 12, the iterative process in solving it is very simple and convergence is very fast. The latter can be strengthened by the possibility of determining the correction required to be applied to the approximate value for starting the next cycle of iterations, as will be described later below. Therefore, the numerical solution of Eq. 12 is especially recommended since, in addition, the programming of the solution process is quite simple and easy and even a hand calculator can be used.

Furthermore, Eq. 13 in conjunction with the diagram G_{12} (Fig. 1) permits a fast determination of the value of friction factor f when, for reasons of comparing or checking, its determination becomes necessary. Indeed, Eq. 5 in conjunction with Eqs. 2, 3 and 11a yields the relation:

$$R = \frac{4}{\pi} Q_D \quad (14)$$

From the data given, for flow problems of type 1 or 2, and the diagram G_{12} (Fig. 1) the values of Q_D , R and R_D are determined and then Eq. 13 yields the value of the friction factor f as:

$$f = 2 \left[\frac{R_D}{R} \right]^2 \quad (15)$$

In addition, this determination enables a better estimation of the value of f and further of the initial value of h_f or D for the numerical solution of Eq. 12.

2.3 The second dimensionless Swamee-Jain equation

Applying the dimensional analysis procedure to Eq. 10, using as repeating variables the roughness height k and the temporary variable $S = g S_f$, three dimensionless variables are obtained defined by the relations:

$$Q_k = \frac{Q}{k^2 \sqrt{g k S_f}} \quad (a) \quad \varepsilon = \frac{k}{D} \quad (b) \quad R_k = \frac{k \sqrt{g k S_f}}{v} \quad (c) \quad (16)$$

On the basis of the extreme values of the dimensional variables of Table 2, the corresponding extreme values of the dimensionless variables of Eqs. 16 are obtained as shown in Table 3.

Table 3. Extreme values of dimensionless variables of Eq.16.

	Q_k	ε	R_k
Minimum value	$1.3 \cdot 10^2$	$3.0 \cdot 10^{-7}$	$3.8 \cdot 10^{-5}$
Maximum value	$1.2 \cdot 10^{18}$	$5.0 \cdot 10^{-2}$	$2.8 \cdot 10^3$

In order to reduce the number of logarithmic cycles in the axes X and Y where the dimensionless variables Q_k and ϵ , respectively, are placed, two new dimensionless variables Q_T and δ are introduced defined by the relations:

$$Q_T \equiv \sqrt{Q_k} = \sqrt{\frac{Q}{\sqrt{g k S_f}}} / k \quad (a) \quad \delta \equiv \sqrt{\epsilon} = \sqrt{\frac{k}{D}} \quad (b) \quad (17)$$

With the introduction of the dimensionless variables of Eqs. 16c and 17 into Eq. 10 one obtains the second dimensionless equation of Swamee-Jain.

$$Q_T^2 = - \frac{\pi}{\sqrt{2} \delta^5} \log \left[\frac{\delta^2}{3.72} + \frac{1.78 \delta^3}{R_k} \right] \quad (18)$$

The nomographic solution of Eq. 18 is depicted in Fig. 2. This nomogram, characterized by the distinctive name Nomogram G_{23} , yields a direct solution to simple pipe flow problems of types 2 and 3.

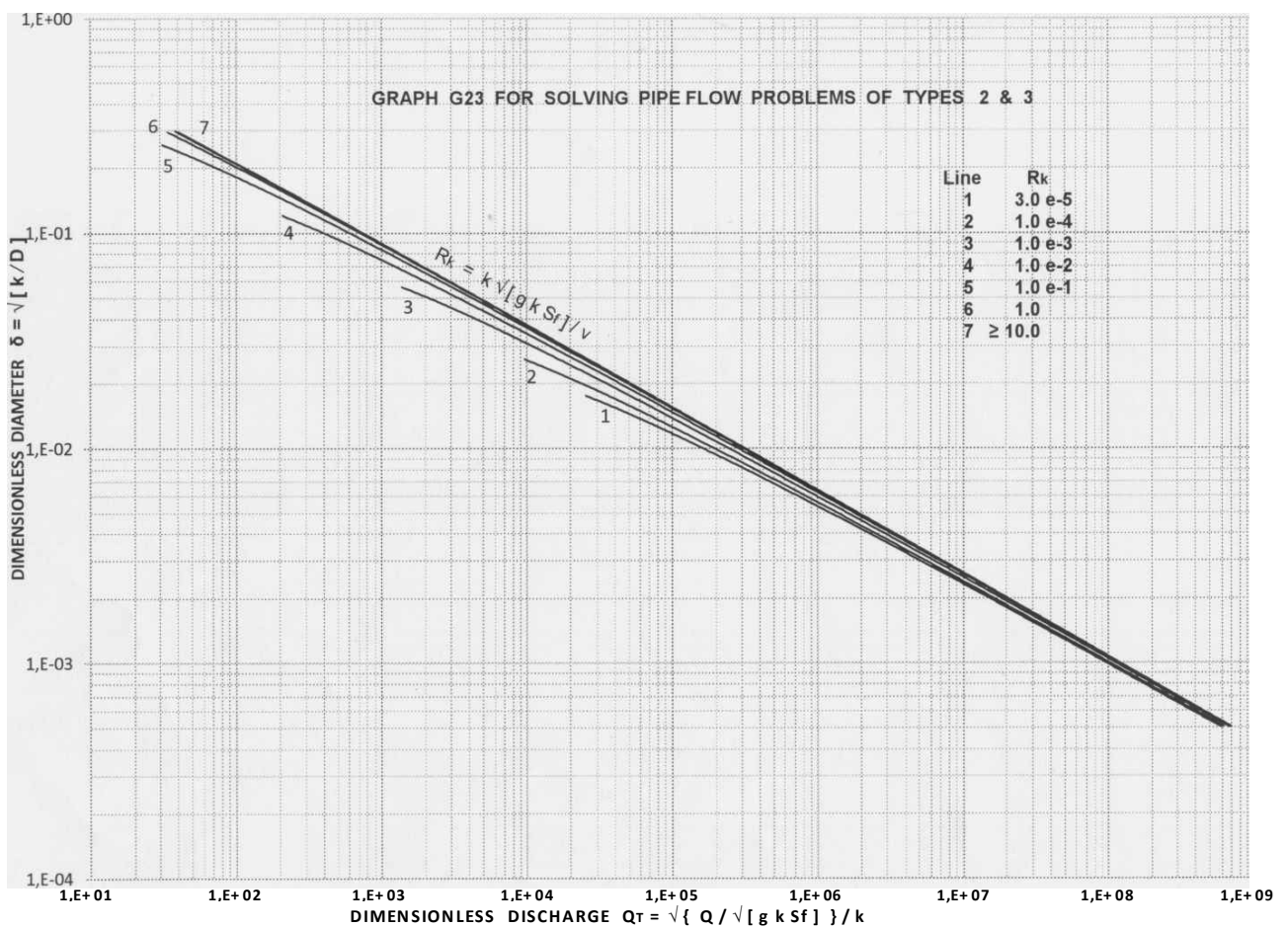


Figure 2. Nomogram G_{23} for the solution of simple pipe flow problems of types 2 and 3.

2.4. Numerical procedure for solving simple pipe flow problems (with Eq. 12)

For the flow problems of type 1 (variable sought the head loss h_f) an adequate first estimation of the head loss h_f or of the energy line slope S_f can be obtained with the following equation 19, which is the result of the combination of Eqs. 1 to 3:

$$h_f = f \frac{8LQ^2}{g\pi^2 D^5} \quad (19)$$

by taking as a first approximation a value $f^{(1)} = 0.020$ (or any other perhaps more accurate according to the judgment of the Designer).

The first application of Eq. 12 (using the first estimated value of head loss $h_f^{(1)}$ which was obtained with the approximate value of $f^{(1)} = 0.020$ as above) yields a first approximation of discharge $Q^{(1)}$ which differs from the given value Q by $\Delta Q^{(1)} = Q - Q^{(1)}$. The correction which must be made in the value $h_f^{(1)}$ is determined by the relation

$$\Delta h_f^{(1)} = \frac{2h_f^{(1)}}{Q} \Delta Q^{(1)} \quad (20)$$

which is derived by differentiating with respect to Q Eq. 19 (under the reasonable and numerically provable assumption that $\partial f / \partial Q \cong 0$) and rearrangement of terms. The second approximation of head loss would be:

$$h_f^{(2)} = h_f^{(1)} + \Delta h_f^{(1)} \quad (21)$$

and the iterative computation goes on in a normal way until the difference $|\Delta Q^{(i)}| = |Q - Q^{(i)}|$ becomes less than the convergence criterion which was set up, usually of the order of 10^{-4} , that is $|\Delta Q^{(i)}| / Q \leq 10^{-4}$.

For the flow problems of type 2 (variable sought the discharge Q), Eq. 12 yields directly the value of the discharge sought.

For the flow problems of type 3 (variable sought the pipe diameter D), an adequate first estimation of the value of the diameter D can be obtained with Eq. 19 (solved for D) by taking again as a first approximation a reasonable value of the friction factor f , usually $f^{(1)} = 0.020$.

The first application of Eq. 12 (using the first estimated value of the diameter $D^{(1)}$) yields a first approximation of the discharge $Q^{(1)}$ which differs from the given value Q by $\Delta Q^{(1)} = Q - Q^{(1)}$. The correction which must be done in the value $D^{(1)}$ is determined by the relation:

$$\Delta D^{(1)} = \frac{2D^{(1)}}{5Q} \Delta Q^{(1)} \quad (22)$$

which is derived by differentiating with respect to Q Eq. 19 (after being solved for D and under the reasonable assumption that $\partial f / \partial Q \cong 0$) and rearrangement of terms. The second approximation of the value of the diameter would be:

$$D^{(2)} = D^{(1)} + \Delta D^{(1)} \quad (23)$$

and the iterative procedure goes on in a normal way until the difference $|\Delta Q^{(i)}| = |Q - Q^{(i)}|$ becomes less than the convergence criterion which was set up, usually of the order of 10^{-4} , that is $|\Delta Q^{(i)}| / Q \leq 10^{-4}$.

The series of computations can be arranged in the form of a table as is shown in the following two examples which were taken from bibliography (Terzidis, 1982: pp. 45, 48).

Example 1. The data given for a flow problem of type 1 are: $k = 2.59 \cdot 10^{-4}$ m, $D = 0.2032$ m (8"), $L = 1000$ m, $Q = 0.130$ m³/s, $\nu = 1.01 \cdot 10^{-6}$ m²/s, $g = 9.81$ m/s². The value of head loss h_f due to wall friction (linear losses) is required.

Solution. For $f = 0.020$ Eq. 19 yields $h_f^{(1)} = 80.616$ m. Further computations, on the basis of Eq. 12, are shown in Table 4 from which one concludes that three iterations are enough for a sufficiently accurate solution of the problem. Hence, $h_f = 85.216$ m.

Table 4. Computation of head losses (type 1 problem).

i	h_f [m]	S_f [*10 ⁻³]	R_D [*10 ³]	Z [*10 ⁻⁶]	$\log Z$	Q_D [*10 ³]	Q [m ³ /s]	ΔQ [*10 ⁻³] [m ³ /s]	Δh_f [m]
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1	80.616	80.616	80.651	364.707	-3.4381	615.967	0.1264	3.584	4.445
2	85.061	85.061	82.845	364.122	-3.4388	632.848	0.1299	0.119	0.156
3	85.217	85.217	82.920	364.102	-3.4388	633.433	0.1300	-0.001	-0.001
4	85.216								

$$\varepsilon = k/D = 1.2746 \cdot 10^{-3}, R_D = D^{3/2} \sqrt{gS_f} / \nu, Z = \varepsilon / 3.72 + 1.78 / R_D, Q_D = (-\pi / \sqrt{2}) R_D \log Z$$

Also, for $Q_D = Q / D \nu = 6.33 \cdot 10^5$ and $\varepsilon = k / D = 1.275 \cdot 10^{-3}$ the diagram G_{12} yields $R_D = 8.20 \cdot 10^4$ and consequently $S_f = (R_D \nu)^2 / g D^3 = 8.334 \cdot 10^{-2}$ or $h_f = 83.34$ m, that is error 2.2%. Moreover, $R = 4 Q_D / \pi = 8.065 \cdot 10^5$ and $f = 2 (R_D / R)^2 = 0.02067$ against the real value $f = 0.02114$.

Example 2. The data given for a flow problem of type 2 are: $k = 3.05 \cdot 10^{-3}$ m, $D = 0.305$ m, $h_f = 6.10$ m, $L = 305$ m, $\nu = 1.01 \cdot 10^{-6}$ m²/s, $g = 9.81$ m/s². Determine the value of the discharge Q .

Solution. One has $\varepsilon = k/D = 0.010$, $S_f = h_f / L = 0.020$, $R_D = D^{3/2} \sqrt{gS_f} / \nu = 7.38717 \cdot 10^4$ and Eq. 12 yields $Q_D = (-\pi / \sqrt{2}) R_D \log(\varepsilon / 3.72 + 1.78 / R_D) = 4.2119 \cdot 10^5$. Consequently, $Q = Q_D D \nu = 0.12975$ m³/s while from the diagram G_{12} one obtains $Q_D = 4.3 \cdot 10^5$ or $Q = 0.1325$ m³/s that is error + 2%. Moreover, $R = 4 Q_D / \pi = 5.3628 \cdot 10^5$ and $f = 2 (R_D / R)^2 = 0.03795$ against the true value $f = 0.03795$.

On the other hand, $\delta = \sqrt{k/D} = 0.10$, $R_k = k^{3/2} \sqrt{gS_f} / \nu = 73.8717$ and Eq. 18 gives $Q_T^2 = 5.7017 \cdot 10^5$. Consequently, $Q = k^{5/2} \sqrt{gS_f} Q_T^2 = 0.12975$ m³/s, while from the diagram G_{23} one obtains $Q_T = 7.40 \cdot 10^2$ or $Q = 0.1246$ m³/s, that is an error of - 4.0 %.

Example 3. The data given for a flow problem of type 3 are: $k = 9.15 \cdot 10^{-4}$ m, $h_f = 15.20$ m, $L = 1520$ m, $Q = 2.84$ m³/s, $\nu = 1.01 \cdot 10^{-6}$ m²/s, $g = 9.81$ m/s². Determine the value of the required diameter D .

Solution. For $f = 0.030$ Eq. 19 yields $D^{(1)} = 1.1486$ m, let it be 1.15 m. Further computations, on the basis of Eq. 12, are shown in Table 5 from which one concludes that three iterations are enough for obtaining a sufficiently accurate solution of the problem although the first approximation in the value of f is by 57% greater than the true value $f = 0.01906$ as one can see from Eq. 19 if the values of all other variables are introduced in it. Hence, $D = 1.049$ m.

Moreover, one has $R = 4 Q_D / \pi = 3.413 \cdot 10^6$ and consequently $f = 2 (R_D / R)^2 = 0.01908$ as compared with the real value $f = 0.01906$.

Also, for $Q_T = [Q / \sqrt{g k S_f}]^{1/2} / k = 1.892 \cdot 10^4$ and $R_k = k \sqrt{g k S_f} / v = 8.583$, the diagram G_{23} yields $\delta = \sqrt{k/D} = 2.95 \cdot 10^{-2}$ and consequently $D = k / \delta^2 = 1.051$ m against the true value $D = 1.049$ m, that is an error of 0.2 %.

Table 5. Computation of the diameter (flow problem of type 3).

i	D [m]	$k/3.72D$ [$\cdot 10^{-6}$]	R_D [$\cdot 10^3$]	Z [$\cdot 10^{-6}$]	$\log Z$	Q_D [$\cdot 10^6$]	Q [m^3/s]	ΔQ [$\cdot 10^{-3}$] [m^3/s]	ΔD [$\cdot 10^{-3}$] [m]
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1	1.150	213.88	382.44	218.54	-3.6605	3.1098	3.612	-772.03	-125.05
2	1.025	239.98	321.79	245.51	-3.6099	2.5805	2.671	168.66	24.35
3	1.049	234.41	333.32	239.75	-3.6202	2.6806	2.841	-0.91	-0.13
4	1.049								

$$R_D = D^{3/2} \sqrt{g S_f} / v, \quad Z = k / 3.72 D + 1.78 / R_D, \quad Q_D = (-\pi / \sqrt{2}) R_D \log Z, \quad Q = D v Q_D$$

From all that have been exposed up to now, one can conclude that the possibilities for solving simple pipe flow problems, using the methodology developed in this paper, can be formulated as shown in Table 6.

Table 6. Possibilities and procedures for solving simple pipe flow problems.

Equation and Diagram	Type of problem	Data given	Data computed dimensionless	Data required		Solution obtained by		
				Dimensionless	Dimensional	Equation		Diagram D
						D	It	
12	1	Q, D	Q_D, ε	R_D	h_f	No	Yes(+)	Yes
G_{12}	2	h_f, D	R_D, ε	Q_D	Q	Yes	---	Yes
18	2	h_f, D	R_k, δ	Q_T	Q	Yes	---	Yes
G_{23}	3	h_f, Q	R_k, Q_T	δ	D	No	Yes(*)	Yes

Notes to Table 6: D = Directly, It = Iteratively, --- = of no meaning.

(+) The simplest and fastest process of linear iteration is that used in Example 1 in which the speed of convergence is by far accelerated using the correction determined by Eq. 20 to the previously found iterative value of h_f .

(*) With the selection of an initial value $D^{(1)}$, through Eq. 19, the corresponding values of $\varepsilon^{(1)}$ and $R_D^{(1)}$ (for Eq. 12) or $\delta^{(1)}$ and $R_k^{(1)}$ (for Eq. 18) are determined and the procedure continues as in type 2 flow problem (determination of the value of $Q_D^{(1)}$ with Eq. 12 or of $Q_T^{2(1)}$ with Eq. 18). The procedure shown in Example 3, where the correction to the previously used value of the diameter is determined and applied, greatly accelerates the speed of convergence.

3. RECAPITULATION AND CONCLUSIONS

The Swamee-Jain equation, which is the result of the combination of Darcy-Weisbach and Colebrook-White equations in order to eliminate the friction factor f , was set in dimensionless form in two different ways using appropriate forms of dimensionless variables. Each of these two dimensionless equations contains three dimensionless variables and, thus, it can be depicted in a unique graph as a set of one-parameter curves.

Both equations yield a direct (analytic) solution to flow problems of type 2 and through an iterative procedure they give solution to the other two types of flow problems. Convergence of the iterative process is very fast (in general three iterations are required to obtain a sufficiently accurate solution). The compact form of the equations, the linearity of the variables in one of them and the possibility of determining the correction required to be applied to the previously determined value of the variable sought in order to proceed to the next iteration, render the numerical procedure of

solution especially applicable even for a simple programmable calculator or one having about ten storage positions.

The nomographs, prepared on the basis of these two equations, each furnish a direct solution to two types of flow problems: The Nomogram G_{12} to flow problems of types 1 and 2, and the Nomogram G_{23} to flow problems of types 2 and 3.

For covering all possible in practice values of the variables Q_D and R_D in Nomogram G_{12} (Fig. 1) 7 and 7 cycles of the logarithmic scale, correspondingly, are required while for the variables Q_T and δ of the Nomogram G_{23} (Fig. 2) are, correspondingly, required 8 and 6 cycles of the logarithmic scale. The distinctness of the Nomograms and the accuracy obtained, at least as a first approximation, is quite satisfactory. Furthermore, the narrow, strip-like form that both diagrams exhibit makes it possible to develop each of them into a few sheets in order to greatly increase resolution. Two or three at maximum sheets with values of R_D or of Q_T by 3 or 4 logarithmic cycles is by far an adequate enlargement to obtain a great accuracy.

Besides, the extensive use of micro-computers or programmable pocket calculators, the use of Eq. 12 or Eq.18 and the compact iterative procedure (where it is required) furnish an easy, fast and accurate solution.

In order to appraise the value of the new algebraic and graphical methodologies developed in this work, one has to compare their application with the classical procedures of using the combination of the Darcy-Weisbach equation with either the Colebrook-White equation or its graphical representation, the well-known Moody diagram.

a) *The case of numerical solution.* In the classical case, the Colebrook - White equation always requires an iterative procedure in order to yield the value of the friction factor f corresponding to given geometric data and flow conditions. The Darcy-Weisbach equation yields a solution for the unknown hydraulic variable (each time after the value of f has been determined) directly, on the one hand, only for the head loss h_f , and by an iterative procedure, on the other hand, for the discharge Q or the diameter D .

In the new methodology, developed in this paper, the use of the Swamee-Jain dimensionless equation eliminates the iterative procedure required for finding f (since this parameter does not appear in the equation) and hence it yields directly the value of the discharge Q and, through an iterative procedure, the value of either the head loss h_f or the diameter D .

b) *The case of using the available Nomograms.* In the classical case, the use of the Moody diagram yields directly the value of the friction factor f , thus eliminating the corresponding iterative procedure required to determine it. The Darcy-Weisbach equation is solved as described above under (a).

In the new methodology, the use of either of the two Nomograms yields directly, in dimensionless form, the value of the corresponding variable sought, that is, the Nomogram G_{12} furnishes the value of Q or h_f and the Nomogram G_{23} yields the value of Q or D .

The simplification in the solution of all three types of simple pipe flow problems is apparent.

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